Physics 2211 ABC
Fall 2014
Test 2

Test form **421**

Name Official Solution Set

Recitation Section (see back of test):_________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

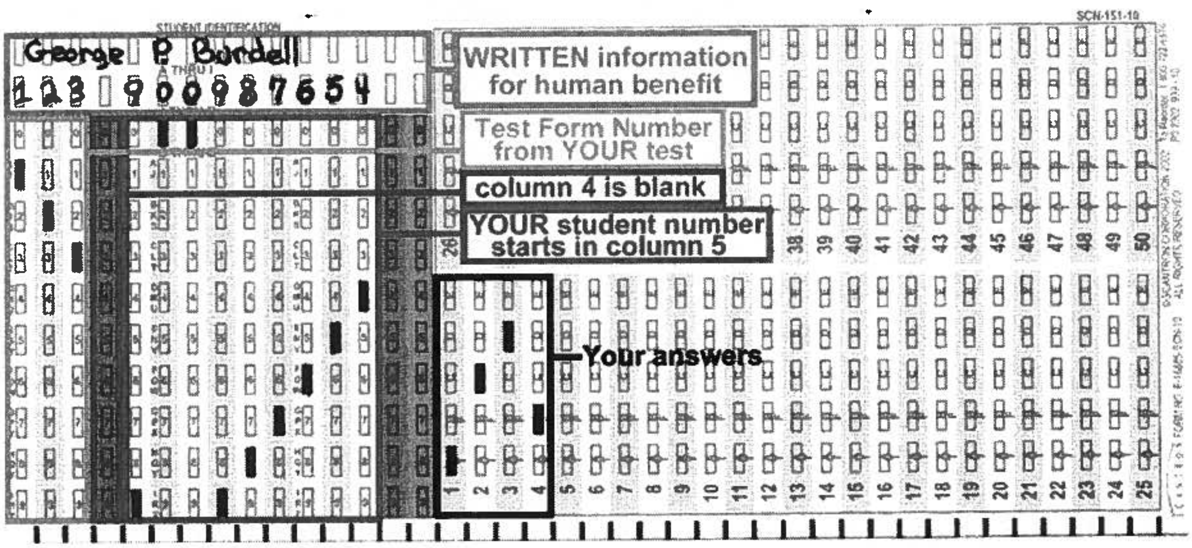
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

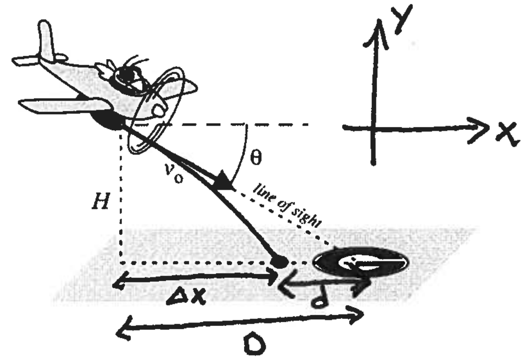
6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

Your test form is: 421



The following problem will be hand-graded. <u>*Show all your work for this problem*</u>. *Make no marks and leave no space on your answer card for it.*

[I] Buzz plans to drop a stink-bomb on the big 'G' at the 50-yardline of Sanford Stadium. He is in a shallow dive at $\theta = 30.0°$ below the horizontal, travelling at a speed $v_o = 75.0$ m/s with the nose of the plane aimed directly at his target. He releases the bomb at a height $H = 49.0$ m above the ground.



(A) *(12 points)* Where does the bomb actually land, relative to the target?

① Decompose initial velocity

$\vec{V}_{ox} = \langle +V_o \cos\theta \rangle$

$\vec{V}_{oy} = \langle -V_o \sin\theta \rangle$

② Determine time of flight, from <u>vertical</u> equation of motion: $\vec{\Delta y} = \vec{V}_{oy}\,\Delta t + \frac{1}{2}\langle -g \rangle \Delta t^2$

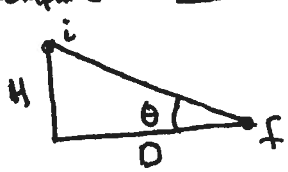
$$\langle -H \rangle = \langle -V_o \sin\theta \rangle \Delta t + \tfrac{1}{2}\langle -g \rangle \Delta t^2$$

$\Rightarrow$ quadratic equation: $\frac{1}{2} g \Delta t^2 + V_o \sin\theta\, \Delta t - H = 0 \rightarrow \Delta t = \dfrac{-V_o \sin\theta \pm \sqrt{V_o^2 \sin^2\theta + 2gH}}{g}$

<u>Positive</u> root is what we want, so $\Delta t = \dfrac{\sqrt{V_o^2 \sin^2\theta + 2gH} - V_o \sin\theta}{g} = \boxed{1.14\,\text{sec}}$

③ Determine <u>horizontal</u> distance traveled during this time: $\vec{\Delta x} = \langle +V_o \cos\theta \rangle \Delta t = \boxed{+73.9\,\text{m}}$

④ Compare to <u>initial</u> distance between bomb and target



$\tan\theta = \dfrac{opp}{adj} = \dfrac{H}{D}$

$D = \dfrac{H}{\tan\theta} = \boxed{84.8\,\text{m}}$

$$\boxed{\text{Bomb falls } \underline{\text{short}}\text{, by a distance } d = D - |\vec{\Delta x}| = 11.0\,\text{m}}$$

(B) *(8 points)* With what velocity does the bomb strike the ground?

$\hookrightarrow$ magnitude and direction relative to horizontal, <u>or</u> paired x- and y- components of velocity vector

① $\vec{V}_{xf} = \vec{V}_{xo} = \langle +V_o \cos\theta \rangle = \langle +65.0\,\text{m/s} \rangle$

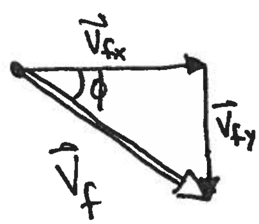
② $\vec{V}_{yf} = \vec{V}_{yo} + \langle -g \rangle \Delta t = \langle -V_o \sin\theta \rangle + \langle -g \rangle \dfrac{\sqrt{V_o^2 \sin^2\theta + 2gH} - V_o \sin\theta}{g}$

$= \left\langle -\sqrt{V_o^2 \sin^2\theta + 2gH} \right\rangle$

$= \langle -48.6\,\text{m/s} \rangle$

So, in cartesian form:

$$\boxed{\vec{V}_f = \langle +65.0\,\text{m/s} \rangle \hat{i} + \langle -48.6\,\text{m/s} \rangle \hat{j}}$$



magnitude: $|\vec{V}_f| = \sqrt{V_{fx}^2 + V_{fy}^2}$

$$\boxed{\begin{array}{l} V_f = 81.2\,\text{m/s} \\ \phi = 36.8° \text{ below horizontal} \end{array}}$$

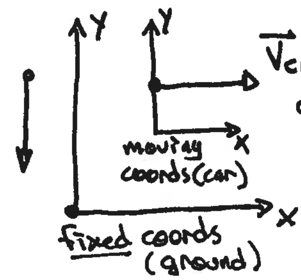
direction: $\tan\phi = \left|\dfrac{V_{fy}}{V_{fx}}\right| = 0.749$

The following problem will be hand-graded. <u>*Show all your work for this problem.*</u> *Make no marks and leave no space on your answer card for it.*

[II] You are driving with a friend in a rainstorm, with a broken speedometer. There is no wind, so the raindrops will fall straight down relative to the Earth. However, when looking out the side window, you notice that the rain appears to be falling at an angle of 70° relative to the vertical. Using your smartphone, you google "terminal speed of rain drops", to discover that a typical randrop falls at a speed of about 22 mph (35 kph). $= "V_2"$ $"V_1" = ??$

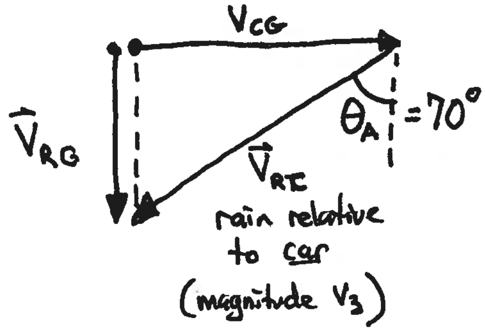
(A) *(12 points)* Assuming that the raindrops fall vertically with a speed of 22 mph, how fast are you driving? Report your answer to two-digit precision, even though that is probably more accurate than you can actually be, in this situation.

we are given velocity of rain wrt ground: $\vec{V}_{RG} = \langle -V_2 \rangle \hat{j}$ rain relative to ground



$\vec{V}_{CG} = \langle +V_1 \rangle \hat{i}$ car relative to ground

we also know: $\vec{V}_{RG} = \vec{V}_{RC} + \vec{V}_{CG}$



$$\langle -V_2 \rangle \hat{j} = \left[\langle -V_3 \sin\theta_A \rangle \hat{i} + \langle -V_3 \cos\theta_A \rangle \hat{j} \right] + \left[\langle +V_1 \rangle \hat{i} \right]$$

remember: θ_A is relative to <u>vertical</u>!

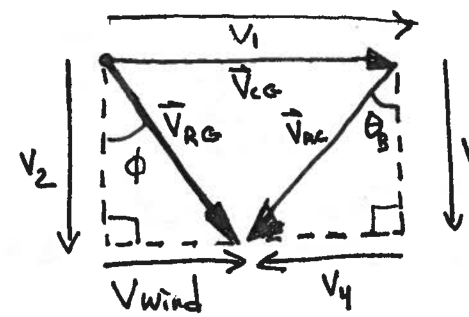
comparing y: $-V_2 = -V_3 \cos\theta_A \rightarrow V_3 = \frac{V_2}{\cos\theta_A}$

comparing x: $0 = -V_3 \sin\theta_A + V_1 \rightarrow V_1 = V_3 \sin\theta_A = \left(\frac{V_2}{\cos\theta_A}\right) \sin\theta_A$

so: $\boxed{V_1 = V_2 \tan\theta_A} = \boxed{60 \text{ mph} = 96 \text{ kph} = 27 \text{ m/s}}$

(B) *(8 points)* A few moment later, a tailwind picks up, and you notice that the raindrops now appear to make an angle of only 60° relative to the vertical. What is the speed of this wind? You may assume that the wind blows <u>horizontally</u>, from <u>directly</u> behind you, so that the <u>vertical</u> speed of the raindrops is still 22 mph. At what angle (relative to the vertical) would a *stationary* observer see the raindrops falling?

$\vec{V}_{CG} = \langle +V_1 \rangle \hat{i} = \langle +V_2 \tan\theta_A \rangle \hat{i}$



Also: from vector addition triangle at right, it is clear that in <u>both</u> reference frames

$\vec{V}_{Rain,y} = \langle -V_2 \rangle$

This allows us to infer x-component of $\vec{V}_{RC}$:

$\tan\theta_B = \left| \frac{V_{RC,x}}{V_{RC,y}} \right| = \left| \frac{V_4}{V_2} \right| \rightarrow \boxed{V_4 = V_2 \tan\theta_B}$

Since $|\vec{V}_{RG,x}| = |\vec{V}_{RG,y}|$, rain falls at angle $\boxed{\phi = 45°}$

Also, x-component of $\vec{V}_{RG} = \vec{V}_{RC} + \vec{V}_{CG}$ is now:

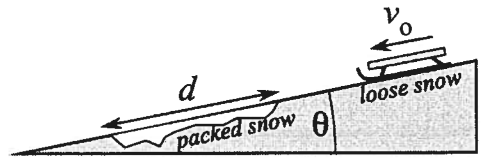
$\langle +V_{wind} \rangle = \langle -V_4 \rangle + \langle +V_1 \rangle$

$V_{wind} = V_2 \tan\theta_A - V_2 \tan\theta_B = V_2(\tan\theta_A - \tan\theta_B) = \boxed{22 \text{ mph}}$
(from A)

horizontal wind speed is <u>equal</u> to vertical rain speed

The following problem will be hand-graded. *<u>Show all your work for this problem</u>.* *Make no marks and leave no space on your answer card for it.*

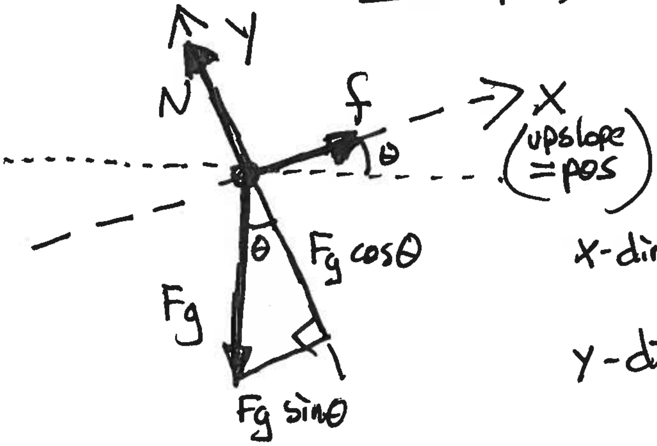
[III] When sled is placed on a hillside covered with loose snow that is inclined at an angle $\theta = 11°$ below the horizontal, it is observed to slide down the hill at constant speed $v_o = 1.5$ m/s.



(A) *(12 points)* Draw a free body diagram for the sled while it is on the loose snow, clearly identifying all forces acting on it. Decompose all forces into components along your chosen coordinate axes, and write out expressions for the 2nd law along each axis.

If the slope were frictionless, the sled would accelerate

→ since we see constant speed, we conclude there IS friction

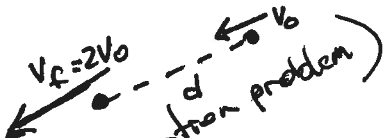


- friction force upslope f
- normal force ⊥ to slope N
- gravitational force F_g

x-direction: $\boxed{\langle +f\rangle + \langle -F_g\sin\theta\rangle = 0}$

y-direction: $\boxed{\langle +N\rangle + \langle -F_g\cos\theta\rangle = 0}$

(B) *(8 points)* The sled hits a patch of packed snow that extends for a distance $d = 2.5$ m along the slope. While on this patch, the sled accelerates, leaving the patch with a speed $2v_o$. <u>Use kinematics to compute the acceleration of the sled</u> while on this patch, and then <u>compare your answer to the value of $g \sin\theta$</u>. Then, draw a free body diagram for the sled while on the packed snow, identifying all forces acting on it. Decompose all forces into components along your chosen coordinate axes, and write out expressions for the 2nd law along each axis.



speed equation for motion along incline

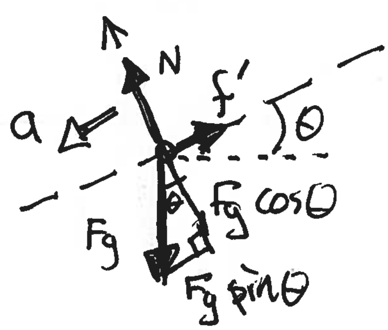
$V_f^2 = V_0^2 + 2\vec{a}\,\vec{\Delta s}$

→ if upslope = positive, $\vec{\Delta s} = \langle -d\rangle$, and $\vec{a}$ must be negatively directed

$(2V_0)^2 = V_0^2 + 2(-a)(-d)$ → $\boxed{a = |\vec{a}| = \frac{3V_0^2}{2d} = 1.35\ \text{m/s}^2}$

compare to $g\sin\theta$ = acceleration in absence of friction: $g\sin\theta = 1.87$ m/s²

since actual a is less than "friction-free" value: $\boxed{\text{there is still friction}}$ (just less of it)



y: $\boxed{\langle +N\rangle + \langle -F_g\cos\theta\rangle = 0}$ (same as before)

x: $\boxed{\langle +f'\rangle + \langle -F_g\sin\theta\rangle = m\langle -a\rangle}$

less than before; non-zero, now

Question value 8 points

(1) A car starts from rest on a circular track of radius R. It accelerates steadily through a half-lap. The magnitude of the car's acceleration is initially a_o. What will be the magnitude of its acceleration as it passes through the quarter-lap position?

$\boxed{\text{(a)}\quad 3.3\, a_o}$

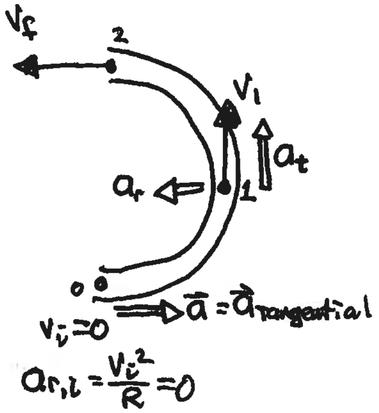
(b) a_o

(c) $2.0\, a_o$

(d) $1.4\, a_o$

(e) $2.8\, a_o$

- Car starts at rest: $V_i = 0$ means initially, no radial acceleration
  $\Rightarrow$ initial acceleration is purely tangential $\vec{a}_{tangential} = \langle +a_o \rangle$
- Uniform tangential acceleration:
  $V_f^2 = \cancel{V_i^2}^{0} + 2\vec{a}_t \cdot \vec{\Delta S}$ $\rightarrow$ for a quarter-circle, $\Delta S = \frac{2\pi R}{4} = \frac{\pi}{2}R$
  so: $V_1^2 = 2a_o\left(\frac{\pi}{2}R\right) = \pi R a_o$
- Since speed is now non-zero, there must be radial accel, to keep turning
  $a_r = V^2/R$ $\rightarrow$ at position 1, $a_{r,1} = V_1^2/R = \frac{\pi R a_o}{R} = \pi a_o$



- At this moment, magnitude of accel vector is found from two components:
  $|\vec{a}_1| = \sqrt{a_{r,1}^2 + a_t^2} = \sqrt{\pi^2 a_o^2 + a_o^2}$
  $\boxed{|\vec{a}| = \sqrt{\pi^2+1}\, a_o = 3.3\, a_o}$

Question value 8 points

(2) A flywheel is constructed as a solid disk of radius 25.0 cm. It is initially spinning at a rate of 12.5 revolutions per second. It experiences a constant angular decceleration of magnitude $\alpha = 1.5$ rad/s^2, which brings the flywheel to a stop. Through what total *distance* will a point on the rim of the flywheel travel, as it comes to a stop?

(a) 65 m

$\boxed{\text{(b)}\quad 514\text{ m}}$

(c) 13 m

(d) 104 m

(e) 227 m

$\omega_i = 12.5\ \frac{\text{rev}}{\text{sec}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} = 25\pi$ rad/sec

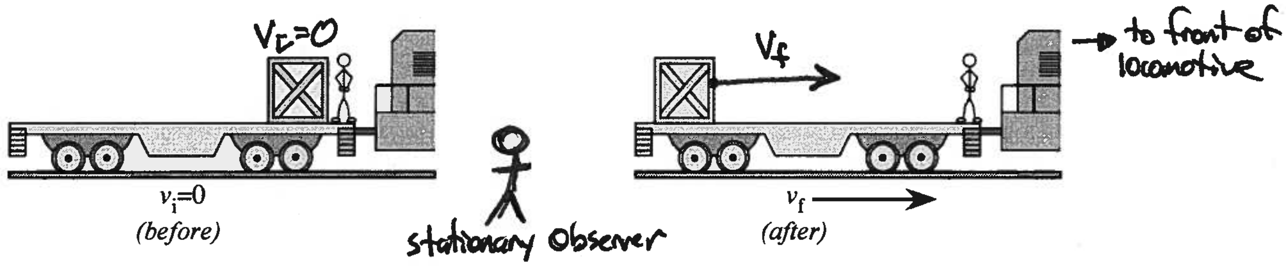
for uniformly accelerated motion:
(with negatively directed angular acceleration, to slow flywheel down)

$\cancel{\omega_f^2} = \omega_i^2 + 2\vec{\alpha}\Delta\theta = \omega_i^2 + 2(-\alpha)\Delta\theta$
0

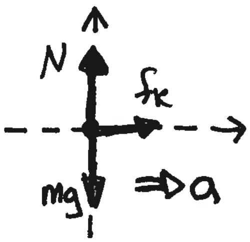
$\Delta\theta = \frac{\omega_i^2}{2\alpha} = \frac{(25\pi\text{ rad/sec})^2}{2(1.5\text{ rad/sec})^2} = \underline{\underline{2056\text{ rad}}} = 327$ rev

convert to tangential distance around rim:
$\Delta S = R\Delta\theta = (0.25\text{m})(2056\text{ rad}) = \boxed{514\text{m}}$

The next two questions involve the following situation:
A railroad flatcar is initially at rest. A large crate lies near the front of the car. A locomotive engine begins pulling the flatcar forward, accelerating uniformly to a final speed v_f. As this is happening, an observer *on the train* sees the crate begin to slip toward the rear of the flatcar, sliding to a stop at the back of the car just as the train reaches its final speed. (The crate does *not* slide all the way off! It begins and ends at rest on the flatcar.)



You will find it advantageous to draw a free body diagram for the crate before you try to answer either question below!



- Flatcar is an accelerating reference frame: can't be used to analyze forces
- Observer ~~on the ground~~ sees crate gain velocity forward/rightward
  Conclusion: rightward/forward acceleration
- Forward accel implies net force = forward
- "backwards" slip relative to car tells us the forward force is kinetic friction

Question value 4 points

(3) What horizontal forces act on the crate, as the train is accelerating?

(a) A forward-directed friction force, and an *equal* backward-directed ~~inertial force~~. no such thing

(b) $\boxed{\text{A forward-directed friction force, only.}}$

(c) A backward-directed friction force, and a *larger* forward-directed ~~inertial force.~~ no such thing

(d) A backward-directed friction force, and a *larger* forward-directed force by the ~~locomotive engine~~.

(e) A forward-directed force by the lo~~comotive engine, only~~. Locomotive is not in contact with the crate, and therefore exerts no direct force on the crate itself!

Question value 4 points

(4) What acceleration (if any) would an observer *on the ground* measure *for the crate*, while the train is accelerating?

(a) A forward-directed acceleration, with a magnitude less than that of the train. not completely accurate: problem is flawed

(b) A bac~~kward~~ directed acceleration, with a magnitude equal to that of the train.

(c) The crate has ~~no acceleration~~, to an observer on the ground.

(d) A forward-directed acceleration, with a magnitude ~~equal to that of the train~~.

(e) A b~~ackward~~ directed acceleration, with a magnitude greater that of the train.

Question value 8 points

(5) At a traffic intersection, Sheila's sedan is *stationary*, Carl's camaro is coasting with *constant* velocity, and Alberto's ambulance is *accelerating*. All three drivers observe Walter juggling water-balloons as he walks along the street at constant speed. For which of the three observers will the balloons obey Newton's laws of motion?

(a) Sheila only

(b) All three of them

(c) Sheila and Carl

(d) Carl and Alberto

(e) Carl only

- Newton's laws work as advertised only in an inertial (that is, non-accelerating) reference frame.

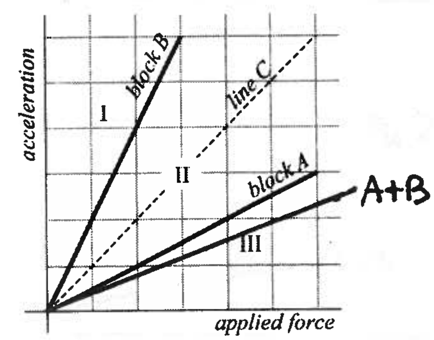
  ⇒ Alberto does not see the ballons as obeying Newton's laws, because he is accelerating

- For observers in different — but inertial — frames, they will all agree about Newton's laws, even if they don't measure the same position/velocity for any of the balloons

Sheila (stationary) and Carl (constant velocity) see Newton's laws work properly. Alberto does not.

Question value 8 points

(6) Two blocks A and B are used in an experiment. Each block is subjected to forces of varying magnitude and the resulting accelerations are measured and plotted in the graph at right. If blocks A and B are glued together and the same experiment is done on the glued pair, where would the graph of a–vs–F lie?



(a) Somewhere above line B, in region I.

(b) Somewhere in region II, but not *necessarily* along line C.

(c) It *must* be *exactly* along line C.

(d) Somewhere below line A, in region III.

(e) There is insufficient information to determine where the graph would be.

Slope A < slope B

$m_A > m_B$

$\vec{a} = \frac{\Sigma \vec{F}}{m}$ ⇒ slope of a-vs-F graph is $\frac{1}{m}$

larger masses have smaller slopes ("shallower" line on graph)

Clearly, when the two objects are glued together, Total mass is even greater than either mass individually

⇒ slope should be more shallow than that of block A

plot will be somewhere in region III