Physics 2211 ABC
Fall 2014
Test 3

Test form 431

Name Official Solutions

Recitation Section (see back of test):________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

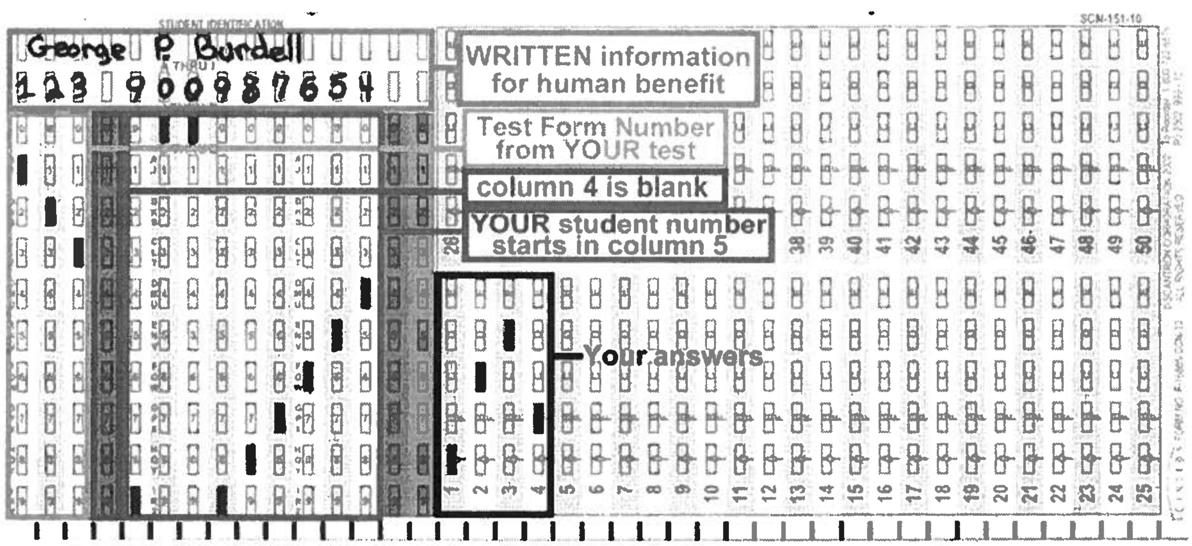
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

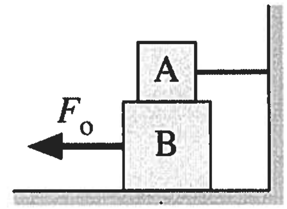
Your test form is: 431



Our next test will be on Monday, October 27!

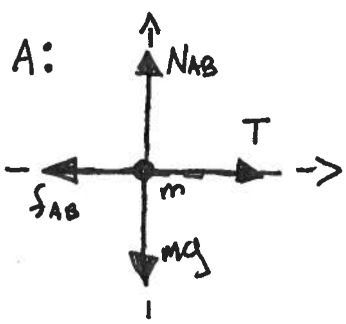
The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[I] Block A, having mass m, is placed on block B, having mass $2m$, which in turn rests on the floor. The surfaces of both blocks and the floor are rough, and all are made of the same material. An ideal cord securely attaches block A to the nearby wall. When block B is pulled to the left by an applied force of magnitude F_0, it remains stationary.

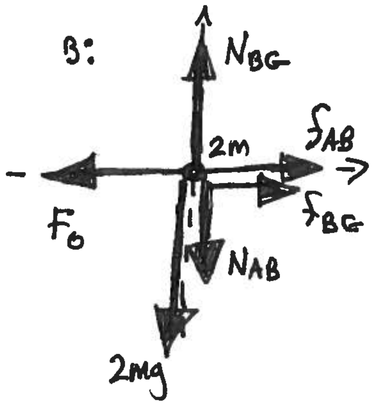


(A) *(8 points)* Draw a free body diagram for each of the blocks, and then identify all Third Law force pairs in your diagram.

A:



B:



3rd law pairs:

- f_{AB} = friction between A + B

  [Since B is being pulled out from under A, f_{AB} is: to right on B, to left on A]

- N_{AB} = normal force between surfaces of A + B

(All other forces are paired with something outside the system (earth, wall, object applying force F_0)

(B) *(12 points)* The force applied to block B is gradually increased, and it is found that block B will begin to slip when the applied force exceeds the value $F_1 = 3mg$ (i.e. when the applied force equals the total weight of the two blocks combined). Determine the coefficient of static friction between the surfaces.

All surfaces are identical, so a single value for μ_s applies to both f_{AB} and f_{BG}

→ since block is on the verge of slipping when $F = 3mg$, we can use $f_s = f_{s\,max} = \mu_s N$

$\rightarrow f_{AB} = \mu_s N_{AB}$

$f_{BG} = \mu_s N_{BG}$

for block A:

y-direction: $\langle +N_{AB} \rangle + \langle -mg \rangle = m\vec{a}_y = 0$

$N_{AB} = mg$

$\longrightarrow f_{AB} = \mu_s mg$

for block B:

y-direction — $\langle +N_{BG} \rangle + \langle -N_{AB} \rangle + \langle -2mg \rangle = 2m\vec{a}_y = 0$

$N_{BG} = N_{AB} + 2mg = mg + 2mg = 3mg$

$\longrightarrow f_{BG} = \mu_s \cdot 3mg$

x-direction — $\langle -F_1 \rangle + \langle +f_{AB} \rangle + \langle +f_{BG} \rangle = (2m)\vec{a}_x = 0$ (block B is not *yet* slipping, so $\vec{a} = 0$)

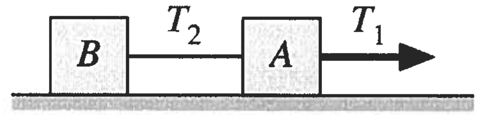
$-3mg + \mu_s mg + \mu_s \cdot 3mg = 0$

$4\mu_s = 3 \longrightarrow \boxed{\mu_s = \frac{3}{4} = 0.75}$

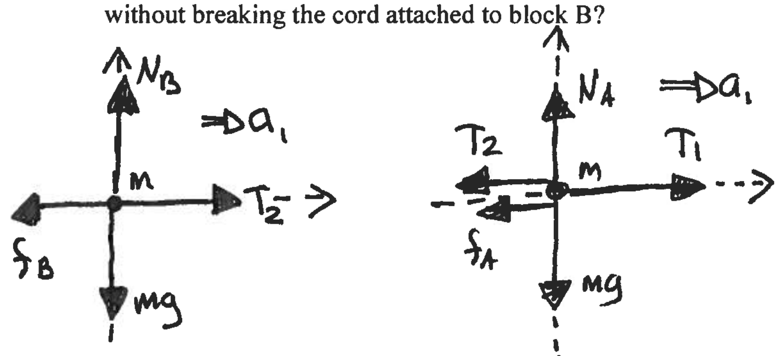
FIX!

The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[II] Blocks A and B have equal masses $m = 4.0$ kg, and are connected via an ideal cord of length $L = 50$ cm. The cord can sustain a maximum tension $T_2 = 24$ N before it will break. The coefficients of kinetic friction for the two blocks are $\mu_A = 0.25$ and $\mu_B = 0.50$.



(A) *(8 points)* What maximum pulling force T_1 can be applied to block A without breaking the cord attached to block B?



Block A: $\sum \vec F_y = 0 = \langle +N_A\rangle + \langle -mg\rangle$

$N_A = mg$

so $f_A = \mu_A N_A = \mu_A mg$

Block B: $\sum \vec F_y = 0 = \langle +N_B\rangle + \langle -mg\rangle$

$N_B = mg$

so $f_B = \mu_B N_B = \mu_B mg$

for B: $\sum \vec F_x = m\vec a_x \rightarrow \langle +T_2\rangle + \langle -f_B\rangle = m\langle +a_1\rangle$

$\boxed{T_2 - \mu_B mg = ma_1}$

for A: $\sum \vec F_x = m\vec a_x \rightarrow \langle +T_1\rangle + \langle -T_2\rangle + \langle -f_A\rangle = m\langle +a_1\rangle$

$\boxed{T_1 - T_2 - \mu_A mg = ma_1}$

→ two equations in unknowns T_1 and a_1 ⇒ solve for T_1!

$$T_2 - \mu_B mg = ma = T_1 - T_2 - \mu_A mg \rightarrow T_1 = 2T_2 - (\mu_B - \mu_A)mg = 2[24\,N] - (0.25)(39.2\,N)$$

$\boxed{T_1 = 38.2\,N}$

to two-digit accuracy: 38N

(B) *(12 points)* If block A is pulled with a force T_3 that is half the pulling force T_1 found in Part A, what will be the tension in the cord connecting block B to block A? [Hint: it is not 12 N!]

Same relationships as in part A, but now, let $T_3 = \frac{1}{2}T_1 = 19.1$ N be the pulling force, and let unknown tension in cord be T_4

⇒ we have $T_4 - \mu_B mg = ma_2 = T_3 - T_4 - \mu_A mg$ [a_2 = new acceleration, but it again drops out, anyway]

$$2T_4 = T_3 + (\mu_B - \mu_A)mg$$

$$T_4 = \frac{1}{2}\left[T_3 + (\mu_B - \mu_A)mg\right]$$

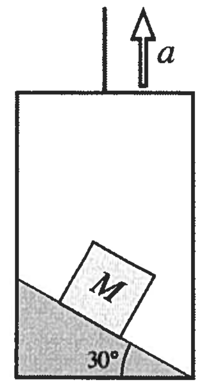
$$= \frac{1}{2}\left[19.1\,N + 9.8\,N\right]$$

$\boxed{T_4 = 14.45\,N}$

to two-digit accuracy: 14N

The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

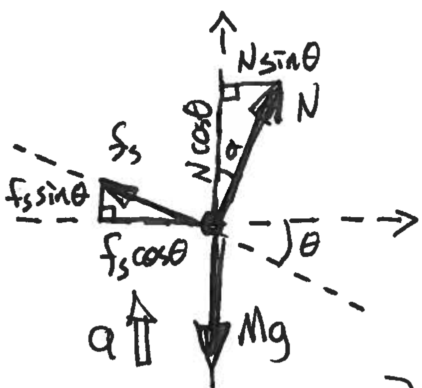
[III] A block of mass $M = 12$ kg rests on a *slanted* elevator floor that makes an angle of 30° below the horizontal. The coefficient of static friction between the block and floor of $\mu_s = 0.75$. When the elevator is accelerating upward with a magnitude $a = 1.5$ m/s^2, the block does not slip along the floor.



In this problem, it will be advantageous to choose coordinate axes based on the acceleration vector, not the slanted floor!

that is, horizontal and vertical axes

(A) *(8 points)* What is the friction force acting on the block?



friction must be upslope, because $\vec{a}_x = 0$, and something has to cancel the x-component of $\vec{N}$

$$\sum \vec{F}_x = m\vec{a}_x = 0$$

$$\langle +N\sin\theta \rangle + \langle -f_s\cos\theta \rangle = 0$$

$$\boxed{N = f_s \cdot \frac{\cos\theta}{\sin\theta}}$$

$$\sum \vec{F}_y = m\vec{a}_y$$

$$\langle +N\cos\theta \rangle + \langle +f_s\sin\theta \rangle + \langle -Mg \rangle = M\langle +a \rangle$$

$$\boxed{N\cos\theta + f_s\sin\theta = M(g+a)}$$

We now have two equations in unknowns N, f_s

$$\left(f_s \frac{\cos\theta}{\sin\theta}\right)\cos\theta + f_s\sin\theta = M(g+a)$$

$$\frac{f_s}{\sin\theta}\left[\cos^2\theta + \sin^2\theta\right] = M(g+a)$$

$$\boxed{f_s = M(g+a)\sin\theta} = \underline{68\,N}$$

(B) *(12 points)* What is the apparent weight of the block as it accelerates upward? (That is, what is the perpendicular force that a spring scaled placed beneath the block would register?)

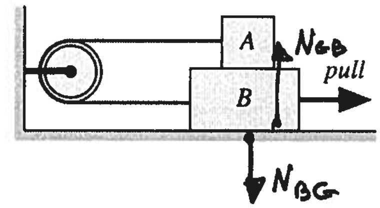
"apparent weight" = force ⊥ to surface acting on block

means: normal force

from above, $N = f_s \cdot \frac{\cos\theta}{\sin\theta} = \left[M(g+a)\sin\theta\right]\frac{\cos\theta}{\sin\theta}$

$$\boxed{N = M(g+a)\cos\theta} = \underline{117\,N}$$

The next two questions involve the following situation:
Wooden block A is stacked on wooden block B, and an ideal cord passes over a pulley to connect the blocks as shown at right. Block B is then pulled to the right with sufficient force to cause it to slip along the ground.



Question value 4 points

(1) According to the Third Law, What force is paired with the upward normal force by the ground on the bottom of block B?

(a) The gravitational force down on block B.

(b) The weight of *both* blocks A and B, down on block B.

(c) A downward normal force by block B on the ground.

(d) The downward normal force by block A on block B

(e) The downward normal force by block B on block A.

By 3rd law
"upward normal by G on B"
pairs with
"downward normal by B on G

Question value 4 points

(2) What pair of horizontal "Third Law" forces act on the two blocks?

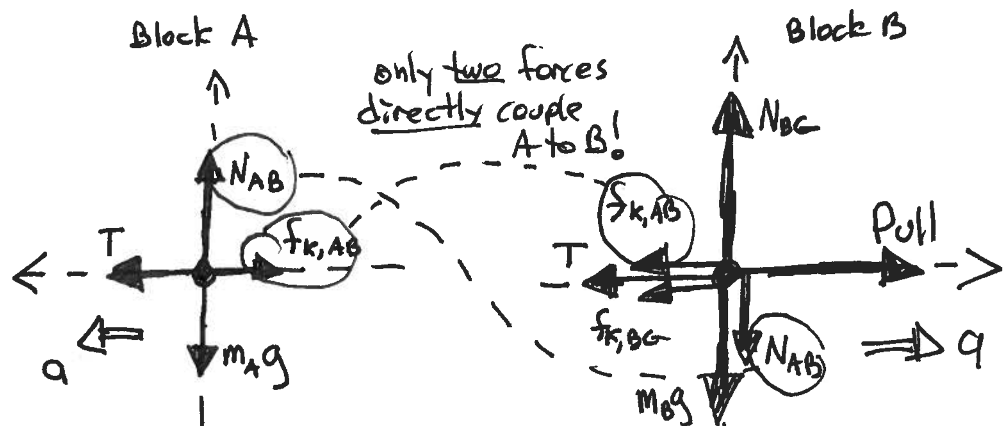
(a) ~~Tension to the left on block B and Tension to the right on block A.~~ Tension acts to left on both blocks

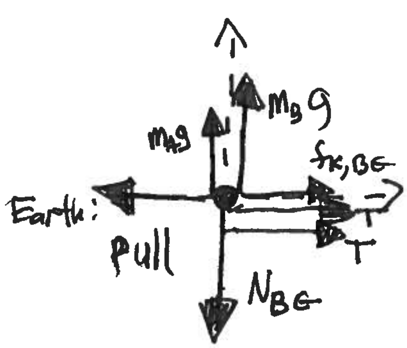
(b) ~~Static friction to the right on block B and to the left on block A.~~ If B moves right, A moves left

(c) ~~Kinetic friction to the right on block B and to the left on block A.~~ A slips left along B → friction would be to the right

(d) Kinetic friction to the left on block B and to the right on block A.

(e) ~~Tension to the left on block B and also to the left on block A.~~ Forces are not oppositely directed

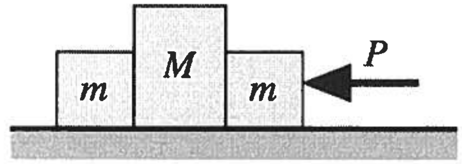




Pulling agent and wall/pulley gizmo are assumed to be attached to, and part of, The Earth

Question value 8 points

(3) Three blocks lie at rest on a $\boxed{\text{frictionless surface}}$. When a horizontal push of magnitude P is applied from the right, what is the magnitude of the net force acting on block M?



$\boxed{\text{(a)}\quad PM/(M+2m)}$

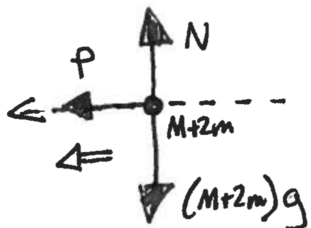
(b) P

(c) $2Pm/M$

(d) $Pm/(M+2m)$

(e) $PM/2m$

① Since surface is frictionless, treat all three blocks as a single system, moving together with acceleration a



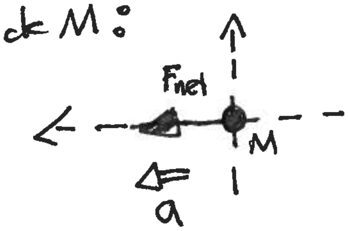
$\Sigma \vec{F}_x = m\vec{a}_x$

$\langle +P\rangle = (M+2m)\langle +a\rangle$

$a = \dfrac{P}{M+2m}$

② Look at only block M:

$\Sigma \vec{F}_y = m\vec{a}_y = 0$



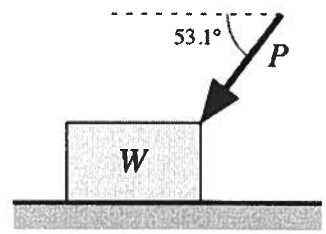
$\underbrace{\Sigma \vec{F}_x} = M\vec{a}_x = M\left\langle +\dfrac{P}{M+2m}\right\rangle$

This IS the net force that was asked for!

(It is the vector sum of a leftward push by the block on the right and a rightward push by the block on the left)

Question value 8 points

(4) A block having weight W = 8 N rests on a horizontal surface where the coefficient of static friction μ_s = 0.50. When an applied force P = 5 N is applied to the block at a 53.1° downward angle as shown, the block does not move. What is the friction force acting on the block at this time?



some useful trig:

cos(53.1°) = 3/5

sin(53.1°) = 4/5

tan(53.1°) = ~~3/4~~ 4/3 (oops!)

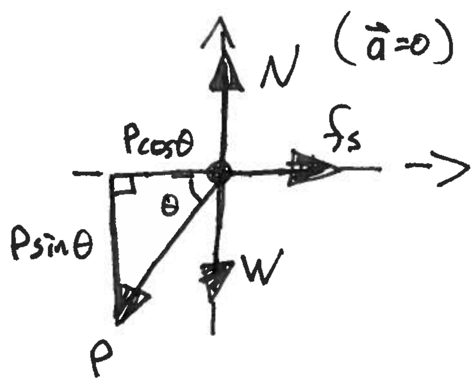
(a) 6 N

$\boxed{\text{(b)}\quad 3\text{ N}}$

(c) 2 N

(d) 4 N

(e) 5 N



Require $\Sigma \vec{F}_x = 0$, since block does not move!

$\langle +f_s\rangle + \langle -P\cos\theta\rangle = 0 \Rightarrow f_s = P\cos\theta = P\left(\tfrac{3}{5}\right)$

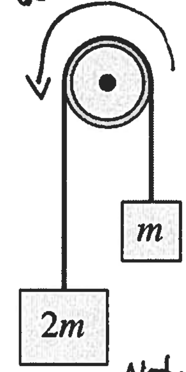
$$\boxed{f_s = 3\text{ N}}$$

(note that $f_{s,max} = \mu_s N$ is larger than this value, but nowhere in The Halls of Hallowed Physics does it say that static friction always takes its maximum possible value...)

Question value 8 points

(5) Two blocks having masses $2m$ and m are hung from a pulley as shown at right. The cord connecting the blocks is massless and unstretchable. When the blocks are released from rest, what will be the tension in the cord?

acceleration!



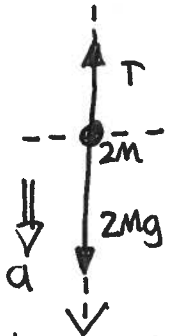
m accelerates up: $T > mg$

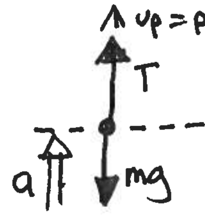
Note: $T < 2mg$ (2m accelerates down)

- ~~(a) 3mg~~
- ~~(b) mg~~
- (c) $\boxed{\frac{4}{3}mg}$
- (d) $\frac{3}{2}mg$
- ~~(e) 2mg~~

only two answers can possibly make sense!

It is critical to realize there will be **acceleration** in this problem!





careful!

$$\langle +2mg \rangle + \langle -T \rangle = (2m)\langle +a \rangle$$

$$\langle +T \rangle + \langle -mg \rangle = m\langle +a \rangle$$

$$T - mg = ma$$

$$2mg - T = 2ma = 2(T - mg)$$

$$4mg = 3T$$

$$\boxed{T = \frac{4}{3}mg}$$

T and a are unknowns ⟹ solve for T

Question Value 8 points

(6) In an amusement park ride called the "Drop of Doom", passengers are strapped into an open seat, and hoisted by a cable to a height H above the ground. The cable is then allowed to go slack and passengers drop in free fall for a distance 3/5 H, at which point the cable is placed under constant tension to brake them to a stop as they fall the final 2/5 H. If a passenger's true weight is represented by W, what will be his *percieved* weight while the tension is slowing him down? [Hint: start by doing some kinematics.]

- (a) 1.67 W
- (b) 2.00 W
- ~~(c) 0.67 W~~
- (d) 1.33 W
- (e) $\boxed{2.50\ W}$

Not consistent with upward accel!

① free-fall through $\Delta y = -\frac{3}{5}H$

$$V_1^2 = \cancel{V_0^2} + 2(-g)(-\tfrac{3}{5}H)$$

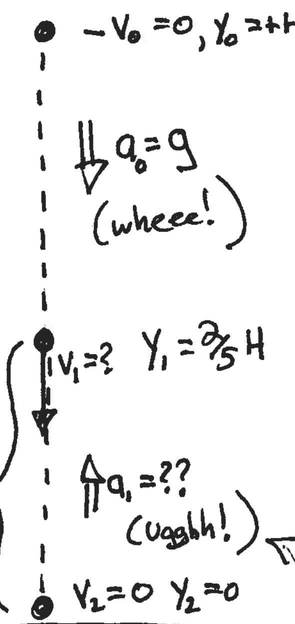
$$V_1^2 = \frac{6}{5}gH$$

② Upward accel under tension:

$$\cancel{V_2^2} = V_1^2 + 2(+a_1)(-\tfrac{2}{5}H)$$

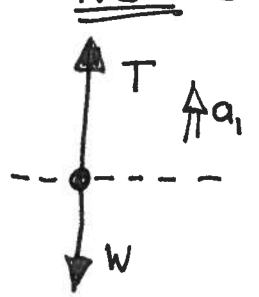
$$\frac{4}{5}a_1 H = \frac{6}{5}gH \implies \boxed{a_1 = \frac{3}{2}g}$$

Two successive kinematics steps!



Upward accel: Tension MUST be greater than grav force!

Now do the 2^{nd} Law problem:



$$\langle +T \rangle + \langle -W \rangle = m\langle +a_1 \rangle \quad \text{where } m = \frac{W}{g}$$

$$T = W + \left(\frac{W}{g}\right)\left(\frac{3}{2}g\right) = W\left[1 + \tfrac{3}{2}\right]$$

$$\boxed{T = 2.5W} = \text{perceived weight}$$