Physics 2211 ABC
Fall 2014
Test 4

Test form 441

Name Official Solutions

Recitation Section (see back of test):_________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

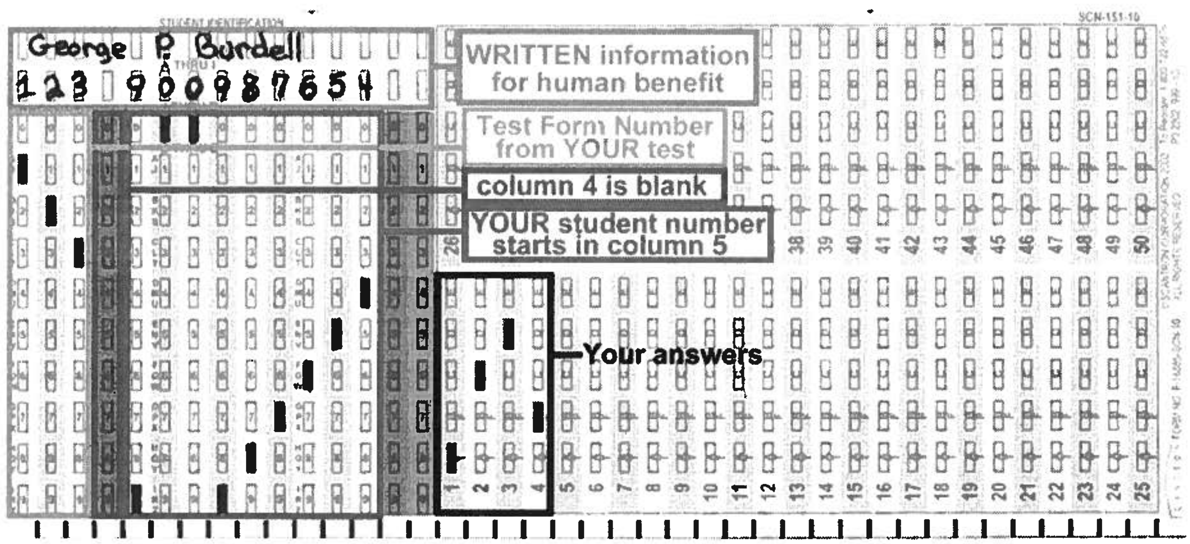
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

Your test form is: 441

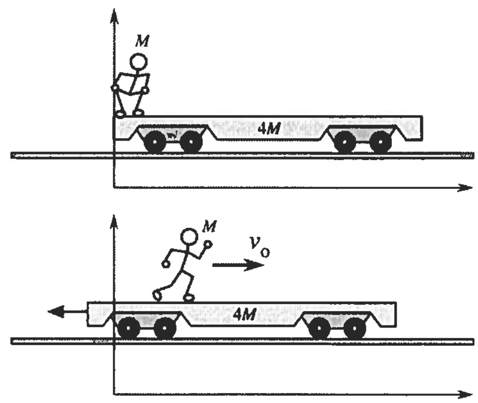


Our next test will be on Monday, November 17!

Our Final Exam will be on Thursday, December 11 [Period Ten, 8:00am]

The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

[I] A sumo wrestler (mass M) is at rest at the very end of a flatcar (mass $4M$, length L) that is initially at rest. He then begins to run toward the other end of the flatcar with a speed v_0, *measured relative to the car, not the ground*.



(A) *(12 points)* Determine the velocity of both the sumo and the flatcar, as measured by an observer on the gound. (You may presume that there is negligible rolling friction between the flatcar's wheels and the ground.)

let V_1 = speed of sumo relative to ground:

$$\vec{V}_{sg} = \langle +V_1 \rangle$$

let V_2 = speed of flatcar relative to ground:

$$\vec{V}_{cg} = \langle -V_2 \rangle$$

- Relative velocity: $\vec{V}_{sg} = \vec{V}_{sc} + \vec{V}_{cg} \Rightarrow \langle +V_1 \rangle = \langle +V_0 \rangle + \langle -V_2 \rangle \Rightarrow \boxed{V_1 = V_0 - V_2}$ or $V_2 = V_0 - V_1$
- Conservation of momentum: $\vec{P}_i = \vec{P}_f \Rightarrow 0 = M\langle +V_1 \rangle + 4M \langle -V_2 \rangle \Rightarrow \boxed{V_1 = 4V_2}$ or $V_2 = \frac{V_1}{4}$

We now have two equations in two unknowns

$$V_1 = 4V_2 = 4(V_0 - V_1) \Rightarrow 5V_1 = 4V_0 \rightarrow V_1 = \frac{4}{5}V_0$$

so $\boxed{\vec{V}_{sg} = \langle +\frac{4}{5}V_0 \rangle}$

yes - to be a vector answer, there must be an explicit positive sign!

and $\vec{V}_{cg} = \langle -V_2 \rangle = \langle -\frac{1}{4}V_1 \rangle$

$$\boxed{\vec{V}_{cg} = \langle -\frac{1}{5}V_0 \rangle}$$

(B) *(8 points)* How far has the flatcar rolled at the moment the sumo reaches the other end of the car? How far is the sumo from his starting position, at that moment? [No, it is *not* simply L!]

Note: sumo moved distance L relative to car, at speed Vo relative to car

→ time required to reach end is found from $\vec{\Delta x}' = \vec{V}\Delta t$

$+L = +V_0 \Delta t \rightarrow \boxed{\Delta t = \frac{L}{V_0}}$

hence, displacement of sumo along ground is $\vec{\Delta X} = \vec{V}_{sg}\Delta t = \langle +\frac{4}{5}V_0 \rangle \cdot \frac{L}{V_0}$

$$\boxed{\vec{\Delta X}_{sumo} = \langle +\frac{4}{5}L \rangle}$$

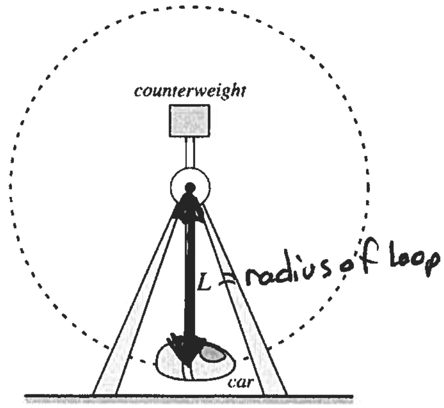
displacement of car is similarly

$$\boxed{\vec{\Delta X}_{car} = \langle -\frac{1}{5}V_0 \rangle \frac{L}{V_0} = \langle -\frac{1}{5}L \rangle}$$

["how far" is a distance question, so scalar answers are okay: $D_s = \frac{4}{5}L$, $D_c = \frac{1}{5}L$]

The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

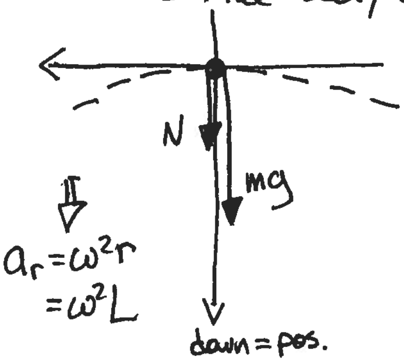
[II] In a carnival ride known as the Loop-O-Plane, passengers are strapped into an enclosed car that is whirled in a vertical circle by a rigid arm of length $L = 5.0$ m. As the car passes through the highest point, passengers are inverted in their seats.



(A) *(12 points)* A passenger riding in the car experiences a perceived weight equal to **half** his true body weight, pushing him *up in to his seat* as he passes through the highest point of his path. At that moment, what is the angular speed of the car? Express your answer in revolutions per second.

Upward perceived weight is actually the result of a normal force down by the seat

⇒ Free-body diagram is:



$$\Sigma \vec{F}_r = m\vec{a}_r$$

$$\langle +N \rangle + \langle +mg \rangle = m\langle +\omega^2 L \rangle$$

$$\tfrac{1}{2}mg + mg = m\omega^2 L \longrightarrow \omega^2 L = \tfrac{3}{2}g$$

worth remembering, to speed up Part B

$$\omega = \sqrt{\frac{3g}{2L}} = \boxed{1.715 \text{ rad/sec}}$$

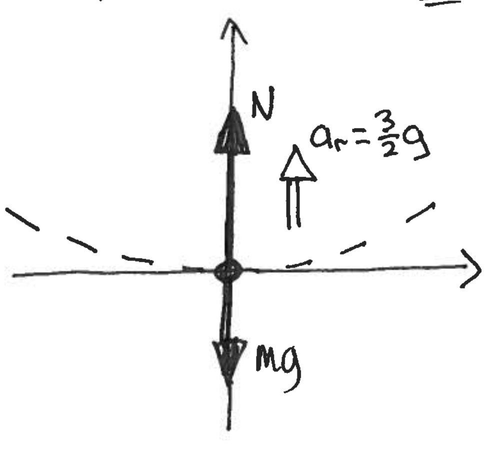
conversion to rev/sec:

$$\omega = 1.715 \text{ rad/sec} \times \frac{1 \text{ rev}}{2\pi \text{ rad}}$$

$$\Rightarrow \boxed{\omega = 0.27 \text{ rev/sec}}$$

(B) *(8 points)* If the car maintains the same angular speed throughout its full arc, what will be the passenger's perceived weight as he passes through the very bottom of the arc? Express your answer as a fraction or multiple of mg.

radial accel is still $a_r = \omega^2 L = \frac{3}{2}g$



$$\Sigma \vec{F}_r = m\vec{a}_r$$

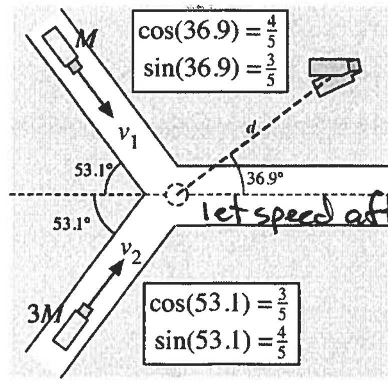
$$\langle +N \rangle + \langle -mg \rangle = m\langle +a_r \rangle = m\langle +\tfrac{3}{2}g \rangle$$

$$\Rightarrow \boxed{N = \tfrac{5}{2}mg}$$

Normal force (up) creates the perception of an apparent weight $\frac{5}{2}mg$, downward

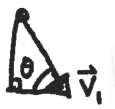
The following problem will be hand-graded. <u>*Show all your work for this problem*</u>*. Make no marks and leave no space on your answer card for it.*

[III] You are a traffic investigator, inspecting the aftermath of a two-truck collision at a Y-intersection. The first truck is unloaded, having mass M. The second truck is loaded, and has a mass $3M$. Just before the collision, a traffic radar system clocked the unloaded truck moving at the posted speed limit, v_1 = 35 mph (15.6 m/s).



let speed afterwards be "V_3"

(A) *(12 points)* Skid marks for the two vehicles indicate that they slid together at an angle of 36.9° north of east. From this information, determine the speed v_2 of the loaded truck just before the collision. Was it exceeding the posted speed limit?



Unloaded car: $\vec{V}_1 = \langle +V_1\cos(53)\rangle\hat{i} + \langle -V_1 \sin 53^\circ\rangle\hat{j}$

$= +\frac{3}{5}V_1\hat{i} - \frac{4}{5}V_1\hat{j}$

loaded car: $\vec{V}_2 = \langle +V_2\cos 53^\circ\rangle\hat{i} + \langle +V_2 \sin 53^\circ\rangle\hat{j} = +\frac{3}{5}V_2\hat{i} + \frac{4}{5}V_2\hat{j}$

Conservation of momentum: $\vec{P}_i = \vec{P}_f = (4M)\left[\langle +V_3\cos 36.9^\circ\rangle\hat{i} + \langle +V_3 \sin 36.9^\circ\rangle\hat{j}\right]$

$$M\left[\tfrac{3}{5}V_1\hat{i} - \tfrac{4}{5}V_1\hat{j}\right] + 3M\left[\tfrac{3}{5}V_2\hat{i} + \tfrac{4}{5}V_2\hat{j}\right] = 4M\left[\tfrac{4}{5}V_3\hat{i} + \tfrac{3}{5}V_3\hat{j}\right]$$

cancel $M/5$ from every term, separate $\hat{i}$ and $\hat{j}$ terms:

$$\boxed{\begin{array}{ll} \hat{i}: & 3V_1 + 9V_2 = 16V_3 \\ \hat{j}: & -4V_1 + 12V_2 = 12V_3 \end{array}}$$

two equations in V_2 and V_3 → eliminate V_3
easiest to divide first by second

$$\frac{3V_1 + 9V_2}{-4V_1 + 12V_2} = \frac{4}{3} \Rightarrow 9V_1 + 27V_2 = -16V_1 + 48V_2$$

$$\Rightarrow 25V_1 = 21V_2$$

$$\boxed{V_2 = \tfrac{25}{21}V_1 = 42\text{mph}}$$

Second driver was speeding

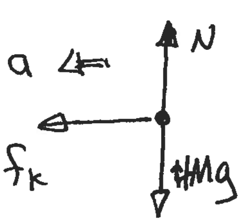
(B) *(8 points)* The skid marks along the pavement extend for a distance d = 9.5 m. What is the coefficient of friction between the trucks' tires and the ground? [Hint: your work in Part A should help you find their speed just *after* they collide.]

① $3V_3 = 3V_2 - V_1 \rightarrow V_3 = V_2 - \frac{1}{3}V_1 = \frac{25}{21}V_1 - \frac{7}{21}V_1 \Rightarrow \boxed{V_3 = \frac{18}{21}V_1 = \frac{6}{7}V_1}$

② Kinematics: $V_f^2 = V_i^2 + 2\vec{a}\Delta\vec{s} \rightarrow 0 = \left(\frac{36}{49}V_1^2\right) + 2(-a)(+d)$

$$a = \frac{36V_1^2}{98d} = 9.41\ \text{m/s}^2 \quad (\text{using } V_1 = 15.6\ \text{m/s})$$

③ Second Law:



$\Sigma F_x = ma_x \rightarrow \langle -f_k\rangle = 4M\langle -a\rangle$ where $f_k = \mu_k N = \mu_k(4Mg)$

$\mu_k 4Mg = 4Ma$

$$\boxed{\mu_k = \frac{a}{g} = 0.960}$$

$\Sigma F_y = 0$

$\langle +N\rangle + \langle -4Mg\rangle = 0 \quad N = 4Mg$

Question value 8 points

(1) A rocket powered sled of mass m = 500 kg generates 12,000 N of thrust. The sled is launched up a frictionless 20° ramp. Which of the paths at right best describes the trajectory of the sled after it leaves the ramp and becomes airborne? (You may assume that the rocket sled continues to aim its nose 20° above the horizontal, after leaving the ramp.)

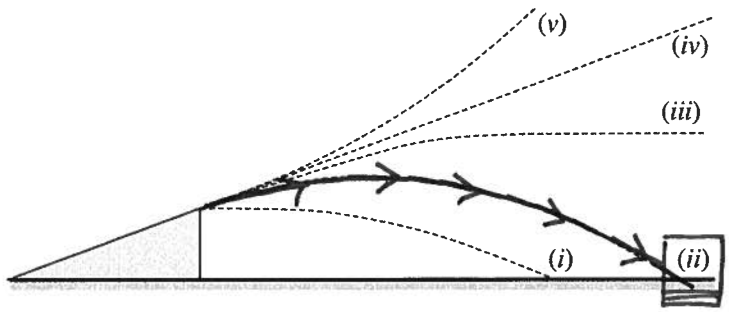
(a) Path (*iii*)

(b) Path (*iv*)

(c) Path (*i*)

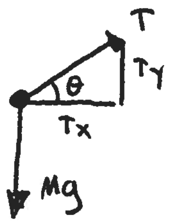
$\boxed{\text{(d) Path } (ii)}$

(e) Path (*v*)



After leaving ramp:

① initial velocity is parallel to ramp



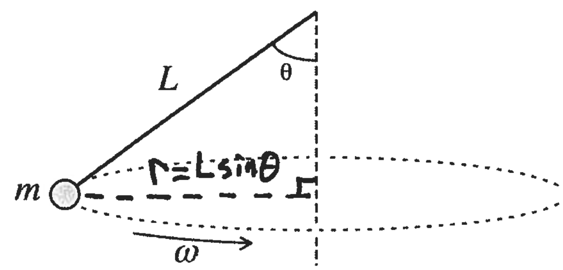
② $\Sigma \vec{F}_y = \langle +T_y \rangle + \langle -Mg \rangle = \langle +T\sin\theta \rangle + \langle -mg \rangle$

$= \langle +4104\,N \rangle + \langle -4900\,N \rangle = \langle -795\,N \rangle$

③ Net vertical force is downward, so trajectory will be downcurving and eventually hit the ground

Question value 8 points

(2) A conical pendulum consists of a cord of length L = 75 cm that can sustain a maximum tension T = 250 N. The pendulum bob has a mass m = 0.50 kg. With what *maximum* angular speed can the bob be rotated without breaking the cord?



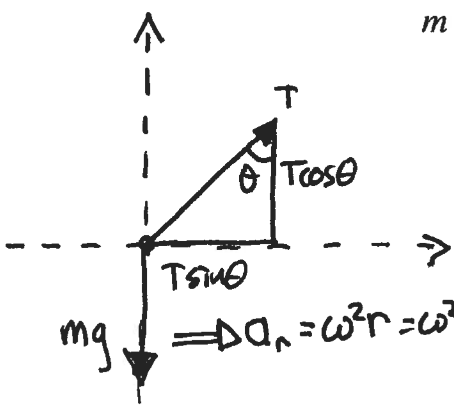
(a) ω_{max} = 2.9 rev/sec

$\boxed{\text{(b) } \omega_{max} = 4.1 \text{ rev/sec}}$

(c) ω_{max} = 3.6 rev/sec

(d) ω_{max} = 5.5 rev/sec

(e) ω_{max} = 4.7 rev/sec



$\Rightarrow a_r = \omega^2 r = \omega^2 L\sin\theta$

$\Sigma \vec{F}_r = m\vec{a}_r$

$\langle +T\sin\theta \rangle = m\langle +\omega^2 L\sin\theta \rangle \rightarrow T_{max} = m\omega_{max}^2 L$

$\omega_{max} = \sqrt{\dfrac{T}{mL}} = 25.8$ rad/sec

$\Rightarrow$ unit conversion

$\omega_{max} = 25.8 \frac{rad}{sec} \times \frac{1\,rev}{2\pi\,rad} = \boxed{4.1 \text{ rev/sec}}$

Question Value 8 points

(3) Tom pushes an empty shopping cart with mass m, applying a force of magnitude F for a time T. Bob pushes a loaded shopping cast with mass $3m$, applying a force of magnitude 4/3 F for a time $T/2$. Both carts begin at rest. What are the relative momenta of the two carts, when Tom and Bob stop pushing?

(a) $p_B = 2p_T$

(b) $p_B = \frac{4}{3}p_T$

(c) $p_B = p_T$

(d) $p_B = \frac{1}{2}p_T$

(e) $\boxed{p_B = \frac{2}{3}p_T}$

Impulse-momentum theorem: $\Delta\vec{p} = \vec{J} = \int \vec{F}dt$

$\vec{p}_f - \underbrace{\vec{p}_i}_{0} = \vec{F}\Delta t$, if force is constant

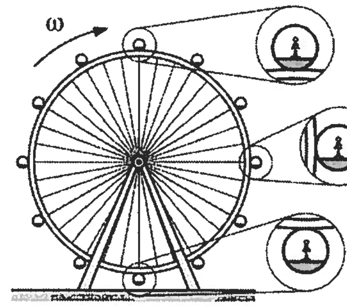
so: $p_T = |\vec{p}_{f,T}| = FT$

$p_B = |\vec{p}_{f,B}| = \left(\frac{4}{3}F\right)\left(\frac{T}{2}\right) = \frac{2}{3}(FT) = \boxed{\frac{2}{3}p_T}$

(mass is irrelevant to this calculation)

Question value 8 points

(4) You are riding in the London Eye, a very large ferris wheel that usually rotates at a leisurely speed of two revolutions per hour. However, a maniac has sabotaged the control mechanism, causing the eye to rotate at two revolutons *per minute*! Rank in order, from greatest to least, your *perceived weight* at the top (W_T), your *perceived weight* at the bottom (W_B), and your *true weight* ($W_o = mg$).



(a) $\boxed{W_B > W_o > W_T}$

(b) $W_B = W_T > W_o$

(c) $W_B = W_o > W_T$

(d) $W_o > W_B = W_T$

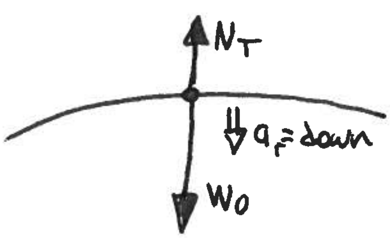
(e) $W_T > W_o > W_B$

① Floor always remains under passenger's seats

⇒ Normal force points up

② perceived weight means "magnitude of normal force"

at the top:



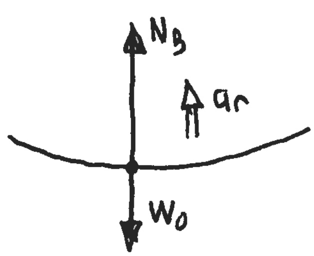
$\Sigma \vec{F}_r = m\vec{a}_r$

$\langle -N_T \rangle + \langle +W_o \rangle = m\langle +\omega^2 R \rangle$

$N_T = W_o - m\omega^2 R$

$\boxed{N_T < W_o}$

at the bottom:



$\Sigma \vec{F}_r = m\vec{a}_r$

$\langle +N_B \rangle + \langle -W_o \rangle = m\langle +\omega^2 R \rangle$

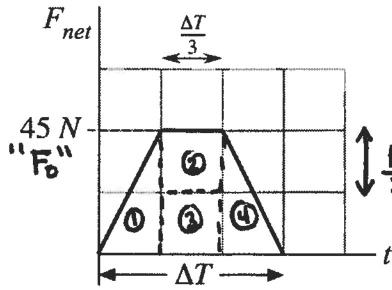
$N_B = W_o + m\omega^2 R$

$\boxed{N_B > W_o}$

Since normal forces create perception of weight,

$\boxed{W_B > W_o > W_T}$

The next two questions involve the following situation:
A basketball of mass $m = 0.650$ kg strikes the ground moving downward with a speed $v_i = 4.00$ m/s, and rebounds upward a few moments later with a speed $v_f = 2.00$ m.s. The graph at right shows the net force acting on the ball while it is in contact with the earth.



Question value 4 points
(5) What is the magnitude of the total impulse delivered to the ball by the earth?

(a) 1.30 N·s
$\boxed{\text{(b) 3.90 N·s}}$
(c) 2.60 N·s
(d) 1.95 N·s
(e) 2.90 N·s

impulse - momentum theorem

$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$

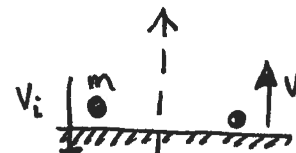
$= m\langle +v_f\rangle - m\langle -v_i\rangle$

$|\vec{J}| = m(v_f + v_i) = 3.9\ \text{kg·m/s}$

$= 3.9\left(\frac{\text{kg·m}}{s^2}\right)s$

equivalent units

$\boxed{J = 3.9\ N\cdot s}$



$\vec{P}_i = m\langle -V_i\rangle$
$\vec{P}_f = m\langle +V_f\rangle$

Question value 4 points
(6) What is the total time ΔT that the ball is in contact with the earth?

(a) 0.33 seconds
$\boxed{\text{(b) 0.13 seconds}}$
(c) 0.04 seconds
(d) 0.21 seconds
(e) 0.09 seconds

definition of impulse:

$\vec{J} = \int \vec{F}dt$ = area under F-vs-t graph

- in graph above, each square has area = height · width = $\left(\frac{\Delta T}{3}\right)\left(\frac{F_0}{2}\right)$
  $\Rightarrow$ 1 square = $\frac{F_0 \Delta T}{6}$
- Area under graph above is clearly four squares = $\frac{4}{6}F_0\Delta T = \frac{2}{3}F_0\Delta T$

So $J = \frac{2}{3}F_0\Delta T \rightarrow \boxed{\Delta T = \frac{3J}{2F_0} = 0.13\ \text{seconds}}$

Question value 4 points extra credit
(7) When is the final exam for this class?

$\boxed{\text{(a) Period Ten — Thursday December 11 at 8:00 am}}$!! People actually missed this !!
(b) Period One — Monday December 8 at 8:00am
(c) Period Seven — Wednesday December 10 at 8:00am
(d) Period Fourteen — Friday December 12 at 11:30am
(e) I don't know because I didn't read the front page of this test.