Physics 2211 ABC
Fall 2014
Test 5

Test form 551

Name Official Solutions

Recitation Section (see back of test):__________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

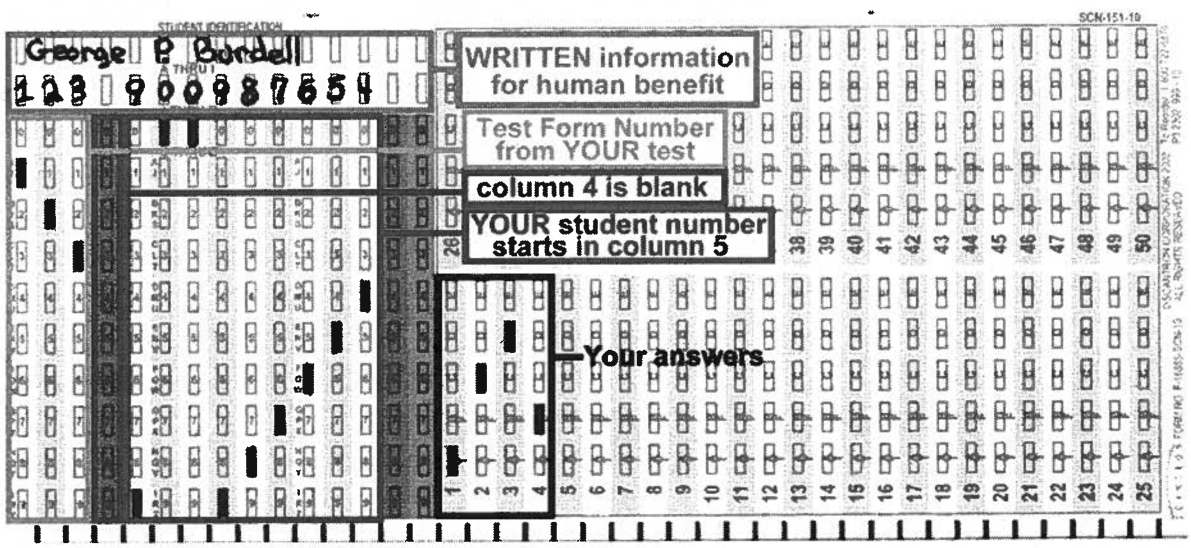
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

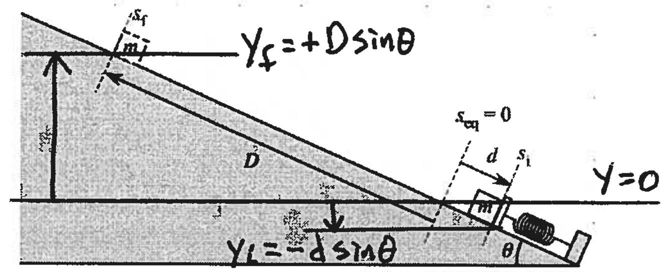
Your test form is: 551



Our Final Exam will be on Thursday, December 11
[Period Ten, 8:00am]

The following problem will be hand-graded. <u>Show all your work for this problem</u>. Make no marks and leave no space on your answer card for it.

[I] A spring having elastic constant k = 480 N/m rests on a rough ramp, inclined at an angle θ = 25° above the horizontal. The spring is compressed by a distance d = 0.10 m, and a block of mass m = 0.25 kg is placed against the spring.



(A) *(12 points)* When the block is released, it is seen to pass through the equilibrium position of the spring with a speed v = 4.2 m/s. What is the coefficient of friction between the block and the ramp?

Please work this problem algebraically, using the symbols v, k, d, m, and θ. Save all numerical substitutions for the final step! ***By doing so, you will significantly improve your chances for partial credit!***

initially: $U_s = \frac{1}{2}k\Delta s^2 = \frac{1}{2}kd^2$ | at equilib: $U_s = 0$

$K = 0$ | $K = \frac{1}{2}mv^2$

$U_g = -mgd\sin\theta$ | $U_g = 0$

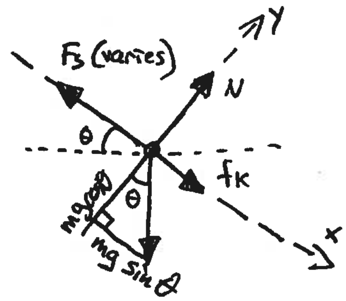
$$W_{nc} = \Delta E \Rightarrow W_{diss} + \cancel{W_{ext}}_{0} = \Delta K + \Delta U_g + \Delta U_s$$

$$-\mu_k mgd\cos\theta = \left(\frac{1}{2}mv^2 - 0\right) + \left(0 - (-mgd\sin\theta)\right) + \left(0 - \frac{1}{2}kd^2\right)$$

$$\mu_k mgd\cos\theta = \frac{1}{2}kd^2 - \frac{1}{2}mv^2 - mgd\sin\theta$$

$$\boxed{\mu_k = \frac{kd^2 - mv^2}{2mgd\cos\theta} - \tan\theta} = 0.413 \text{ rounds to } \boxed{0.41} \text{ (two-digit precision)}$$

or: $\mu_k\cos\theta + \sin\theta = \dfrac{kd^2 - mv^2}{2mgd}$



$\Sigma F_y = m\bar{a}_y = 0$

$\langle +N \rangle + \langle -mg\cos\theta \rangle = 0$

$N = mg\cos\theta$

$\Rightarrow \boxed{f_k = \mu_k N = \mu_k mg\cos\theta}$

then $W_{diss} = -f_k \cdot s$

where s = distance travelled

(B) *(8 points)* How high up the ramp will the block travel before coming to a stop? Express your answer as a distance D measured from the spring's equilibrium position.

For <u>full</u> trip, use K_i, U_{gi}, U_{si} as above, with $K_f = 0$, $U_{sf} = 0$, $U_{gf} = +mgD$

also, total distance of slide is $s = D + d$, so $W_{diss} = -\mu_k mg(D+d)\cos\theta$

$$\Rightarrow -\mu_k mg(D+d)\cos\theta = (0-0) + \left[(+mgD\sin\theta) - (-mgd\sin\theta)\right] + \left(0 - \frac{1}{2}kd^2\right)$$

$$+\mu_k mg(D+d)\cos\theta = -mg(D+d)\sin\theta + \frac{1}{2}kd^2$$

$$mg(D+d)\left[\mu_k\cos\theta + \sin\theta\right] = \frac{1}{2}kd^2$$

$$\cancel{mg}(D+d)\left(\frac{kd^2 - mv^2}{\cancel{2mg}d}\right) = \frac{1}{\cancel{2}}kd^2$$

$$D + d = \frac{d}{kd^2 - mv^2} \cdot kd^2 = \frac{kd^2}{kd^2 - mv^2} \cdot d$$

$$D = \left[\frac{kd^2}{kd^2 - mv^2} - 1\right]d = \left[\frac{kd^2}{kd^2 - mv^2} - \frac{kd^2 - mv^2}{kd^2 - mv^2}\right] = \boxed{\frac{mv^2}{kd^2 - mv^2}d} = 1.13\,\text{m}$$

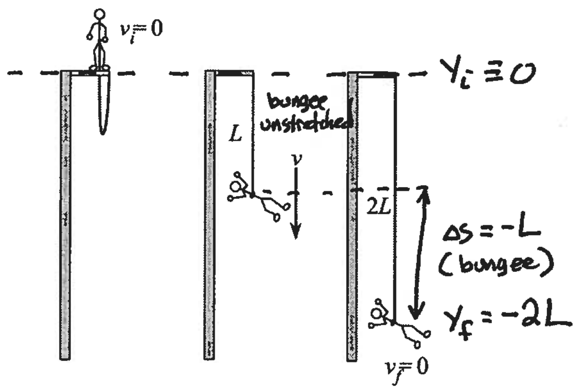
$\boxed{\text{rounds to } 1.1\,\text{m}}$

[Yeah — lots of algebra — but <u>very</u> <u>little</u> calculator work!

(<u>no</u> chance for rounding errors

The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

[II] A bungee cord has an unstretched length L. A jumper of mass m steps off a high platform, and free falls for a distance L, at which point the bungee cord becomes taut and begins to stretch. The jumper is observed to come to a stop a distance $2L$ below the platform.



(A) *(12 points)* What is the spring constant k for the bungee cord? (Express your answer in terms of m, g, and/or L.)

let $Y_i = 0$ at platform

$\Rightarrow K_i = 0,\ U_{gi} = 0,\ U_{si} = 0$

(bungee can't compress, only stretch)

at lowest point:

$K_f = 0,\ U_{gf} = -2mgL\ \ U_{sf} = \frac{1}{2}k(\Delta s)^2 = \frac{1}{2}kL^2$

since no dissipative forces act $E_i = E_f$

$$K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf}$$
$$0 + 0 + 0 = 0 + (-2mgL) + (\tfrac{1}{2}kL^2)$$
$$\tfrac{1}{2}kL^2 = 2mgL$$
$$k = \frac{2mgL}{\frac{1}{2}L^2} = \boxed{\frac{4mg}{L}}$$

Note that expression has units of force $[mg]$ over length $[L]$ → appropriate units for spring constant!

(B) *(8 points)* At what position (measured below the platform) will the bungee jumper attain maximum speed?

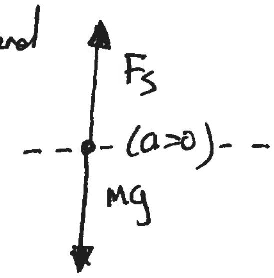
Two possible approaches

① At max speed, acceleration changes from negative/downward to positive/upward

↳ $\vec{a} = 0$ for an instant $\Rightarrow \Sigma F_y = 0$: a moment of equilibrium

$$\langle +F_s \rangle + \langle -mg \rangle = 0 \Rightarrow \langle -k\Delta s \rangle + \langle -mg \rangle = 0$$
$$\rightarrow \Delta s = -\frac{mg}{k} = -\frac{mg}{4mg/L} = \frac{-L}{4}$$

relative to platform, $\boxed{\vec{Y}_{max} = \langle -\frac{5L}{4} \rangle}$



② Energy: for positions y below $-L$, we have: $E(y) = E_i = 0$

$$K(y) + mgy + \tfrac{1}{2}k(y+L)^2 \rightsquigarrow K(y) = -mgy - \tfrac{1}{2}k(y+L)^2$$

(note: bungee has $\Delta s = y - y_{eq} = y - (-L) = y + L$)

maximize K to maximize speed

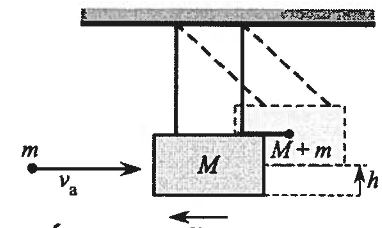
$$\left.\frac{dK}{dy}\right|_{y_m} = -mg - k(y_m + L) = 0$$
$$-mg - \frac{4mg}{L}(y_m + L) = 0$$

$4mg(y_m + L) = -mgL$

$4y = -5L \Rightarrow \boxed{y_m = -\frac{5}{4}L}$

The following problem will be hand-graded. *<u>Show all your work for this problem</u>.* *Make no marks and leave no space on your answer card for it.*

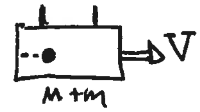
[III] In the ballistic pendulum shown at right, a bullet of mass m = 75 g is moving to the right with a speed v_a= 120 m/s when it strikes a swinging block of mass M = 1.5 kg, just as the block reaches the bottom of its arc and is moving left with a tangential speed v_b = 2.0 m/s. The bullet embeds itself in the block, and the two masses swing upward together to a final height h = 0.48 m above their starting height.



(A) *(12 points)* How much work was done by collisional forces, <u>during</u> the process of the bullet embedding itself in the block? **Be sure to include the <u>sign</u> of the work!**

before

after



In collision, momentum is conserved [and collision is <u>perfectly</u> <u>inelastic</u>]

$$\langle +mV_a\rangle + \langle -MV_b\rangle = (M+m)\langle +V\rangle \quad \rightarrow V = \frac{mV_a - MV_b}{M+m} = \boxed{3.81 \text{ m/s}}$$

To <u>infer</u> work by collision forces, apply Energy Principle: $W_{non\text{-}cons.} = \Delta E_{mech}$

→ negligible motion <u>during</u> collision means $\Delta U \approx 0$ → only ΔK matters

$$W_{coll} = \Delta K = K_f - K_i = \tfrac{1}{2}(M+m)V^2 - \left(\tfrac{1}{2}mV_a^2 + \tfrac{1}{2}MV_b^2\right)$$

$$= (11.43\text{ J}) - (540\text{ J} + 3\text{ J})$$

$$\boxed{W_{coll} = -532\text{ J}}$$ or -530 J, to two-digit accuracy

(B) *(8 points)* How much work was done by aerodynamic drag as the bullet and block rose to their final height, <u>after</u> the collision? **Be sure to include the <u>sign</u> of the work!**

<u>afterwards</u>, again apply energy principle, but now, $K_{after} \rightarrow U_{gf}$

$$W_{drag} = \Delta E_{mech} = \Delta K + \Delta U = \left(0 - \underbrace{\tfrac{1}{2}(M+m)V^2}_{K_f}\right) + \left([M+m]gh - \underbrace{0}_{U_i}\right)$$

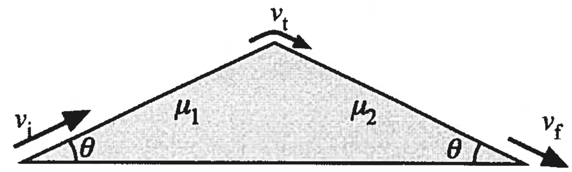
$$W_{drag} = -\tfrac{1}{2}(M+m)V^2 + (M+m)gh$$

$$= -11.43\text{ J} + 7.41\text{ J}$$

$$\boxed{W_{drag} = -4.02\text{ J}}$$ or -4.0 J, to two-digit accuracy

Question value 8 points

(1) A block having mass m and initial speed v_i slides up one side of a double-ramp, having coefficient of friction μ_1. When it reaches the top it transitions smoothly (with no loss of speed) to the other side, where the cofficient of friction is μ_2. It then travels down to the bottom of the ramp, reaching a final speed v_f. Which is larger, the time Δt_1 to ascend the first side of the ramp or the time Δt_2 to descend the second side? [Hint: start by thinking about the *average speeds* on each side of the ramp…]



(a) $\Delta t_2 = \Delta t_1$

(b) The ramp with the greater value of μ takes the longest time.

(c) $\Delta t_1 > \Delta t_2$

(d) The ramp with the lesser value of μ takes the longest time.

(e) $\boxed{\Delta t_2 > \Delta t_1}$

① Friction is present → mechanical energy is lost → when block returns to bottom of ramp, it will have less KE than when it started up the ramp

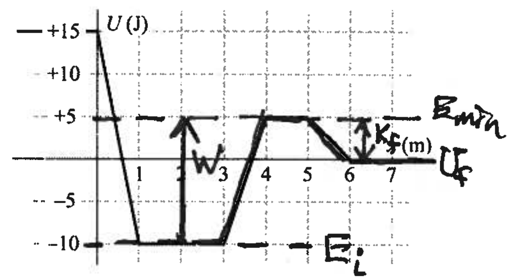
We conclude $V_f < V_i$

② Per the hint, compare average speeds : $V_{up} = \frac{V_i + V_t}{2}$, $V_{down} = \frac{V_t + V_f}{2}$ ⇒ $\boxed{V_{down} < V_{up}}$

③ Since ramps have same length, $V_{down} < V_{up}$ implies $\boxed{\Delta t_2 > \Delta t_1}$

Question value 8 points

(2) A particle is subject to the potential energy function shown at right. It begins at rest at $\vec{x} = \langle +2\text{ m}\rangle$. What *minumum* external work would allow the particle to escape from its location, and where will it end up? (Below, read "≳" to mean, "…just a little bit more than…")



(a) If $W_{ext} \gtrsim$ 25 J, the particle will end up at $-\infty$, with $K_f \gtrsim$ 0 J.

(b) If $W_{ext} \gtrsim$ 10 J, the particle will end up at $+\infty$, with $K_f \gtrsim$ 5 J.

(c) $\boxed{\text{If } W_{ext} \gtrsim 15\text{ J, the particle will end up at } +\infty\text{, with } K_f \gtrsim 5\text{ J.}}$

(d) It is physically impossible for this particle to escape--to do so, it would have to end up with *negative* kinetic energy, which is not allowed.

(e) If $W_{ext} \gtrsim$ 5 J, the particle will end up at $+\infty$, with $K_f \gtrsim$ 0 J.

At rest at $\vec{x} = \langle +2m \rangle$ → $K_i = 0$ and $U_i = -10\text{J}$ ⇒ $E_i = -10\text{J}$

Minimum escape energy requires particle to get "over the hump" between $x = 4m$ and $x = 5m$. This will be possible if total energy is $E \gtrsim 5\text{J}$

[Just barely more than 5J allows $U = 5\text{J}$, K = small and positive]

⇒ $\Delta E = E_{min} - E_i = (+5\text{J}) - (-10\text{J}) = +15\text{J}$ so $\boxed{W_{ext} \gtrsim 15\text{J}}$

Once past the hump, U drops to zero, so $\boxed{K_f \to 5\text{J}}$

Question Value 8 points

(3) An F-18 Hornet (m = 24,000 kg) on an aircraft carrier is catapulted from rest to takeoff speed v = 67 m/sec (~150 mph or 240 kph) in a distance of 82 m (~270 ft). Assume the jet experiences constant acceleration during this launch, and that the jet's engine thrust is negligible in comparison to the catapult force. Which is true about the catapult power used to accelerate the plane?

(a) The average catapult power during the entire launch is 44 MW, which is twice the power at the instant of takeoff.

(b) The average catapult power during the entire launch and the power at the instant of takeoff are both 44 MW.

(c) The catapult power at the instant of takeoff is 44 MW, which is twice the average power during the entire launch.

(d) The average catapult power during the entire launch is 22 MW, which is twice the power at the instant of takeoff.

(e) The average catapult power during the entire launch and the power at the instant of takeoff are both 22 MW.

First, some kinematics: $v_i \doteq 0 \rightarrow v_f = 67 m/s$ in distance $d = 82 m$

$v_f^2 = v_i^2 + 2a\Delta x \longrightarrow v^2 = 2ad \rightarrow \boxed{a = \frac{v^2}{2d}}$ and time to launch is found from

$\Delta v = a\Delta t \rightarrow \boxed{\Delta t = \frac{v}{a} = \frac{2d}{v}}$

Average Power: $P_{av} = \frac{W}{\Delta t} = \frac{\Delta K}{\Delta t} = \frac{\frac{1}{2}mv^2}{2d/v} = \boxed{\frac{mv^3}{4d}}$

Instantaneous power: $P_{INST} = \vec{F}\cdot\vec{v} = (ma)(v) = \left(m\frac{v^2}{2d}\right)v = \boxed{\frac{mv^3}{2d}}$

Calculations are NOT NEEDED: only one answer above has $\boxed{P_{INST} = 2\,P_{av}}$

Question value 8 points

(4) In an alternate universe, scientist Isabella Newton conducts an experiment on gravity. She finds that she can carry a 1.0 kg stone 6.0 m in the horizontal direction and then lift it 4.0 m high, doing a total of 19.6 J of work in the process. She then returns the stone to its starting place by first moving the stone 6.0 m in the horizontal direction and then lowering it 4.0 m vertically, again doing 19.6 J of work in the process. What does Ms. Newton conclude about gravity in her universe?

(a) Gravity is conservative, but stronger in her universe than in ours.

(b) Gravity is conservative, obeying the formula $\Delta U_g = mg(\Delta y)^2$

(c) Gravity is not a conservative force.

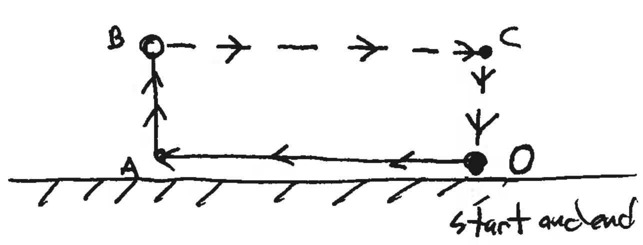
(d) Gravity is conservative, but weaker in her universe than in ours.

(e) Gravity is conservative, obeying the formula $\Delta U_g = -mg^2\Delta y$

$K_i = K_f = 0$, so $W_{TOT} = 0$

but $W_{TOT} = W_{Newton} + W_{grav}$

$\Rightarrow W_{grav} = -W_{Newton}$



Path O→A→B:

$W_{grav} = -W_{Newton} = -19.6 J$

Path B→C→O:

$W_{grav} = -W_{Newton} = -19.6 J$

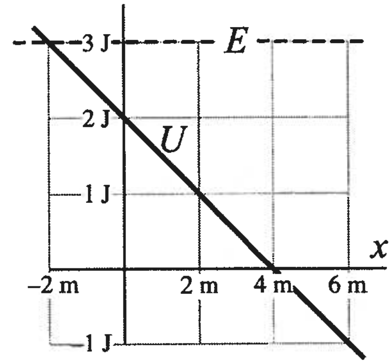
End result: on a closed path, gravity has done net negative work

but if gravity is conservative, $\Delta U_g = -W_{grav} = +39.2 J$

We would conclude that position O has 39 J more PE than position O!!

$\Rightarrow$ self-contradictory conclusion $\rightarrow$ $\boxed{\text{this gravity is NOT Conservative}}$

The next two questions involve the following situation:
A particle of mass $m = 1.0$ kg moves in one dimension, subject to the potential energy function (heavy line) plotted at right. Its total mechanical energy is also indicated by the dashed line.



Question value 4 points

(5) What force will the particle experience when it is at $\vec{x} = \langle +2 \text{ m} \rangle$?

(a) $\vec{F_x} = \langle -2.0 \text{ N} \rangle$

(b) $\vec{F_x} = \langle 0 \text{ N} \rangle$

(c) $\vec{F_x} = \langle -0.5 \text{ N} \rangle$

(d) $\vec{F_x} = \langle +2.0 \text{ N} \rangle$

(e) $\vec{F_x} = \langle +0.5 \text{ N} \rangle$

Force is found as the "gradient" of PE:

$$\vec{F}_x = \left\langle -\frac{dU}{dx} \right\rangle = \left\langle -\frac{\text{rise}}{\text{run}} \right\rangle = \left\langle -\frac{(-2J)}{(+4m)} \right\rangle$$

$$\boxed{\vec{F}_x = +0.5 \text{ J/m} = \langle +0.5 \text{ N} \rangle}$$

Question value 4 points

(6) Where in the region shown will the particle be in equilibrium?

(a) At $\vec{x} = \langle +2 \text{ m} \rangle$

(b) Nowhere in the region shown.

(c) At $\vec{x} = \langle +4 \text{ m} \rangle$

(d) At $\vec{x} = \langle 0 \text{ m} \rangle$

(e) At $\vec{x} = \langle -2 \text{ m} \rangle$

"Equilibrium" means "zero net force"
if force = negative of slope of PE,
then $\vec{F} = 0$ only at locations
where PE is flat (zero slope)
$\Rightarrow$ this condition obviously does not hold anywhere in the region shown