Physics 2211 BCD — Test form 433 — Name Official Solution Set

Spring 2015

Test 3 — Recitation Section (see back of test):__________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

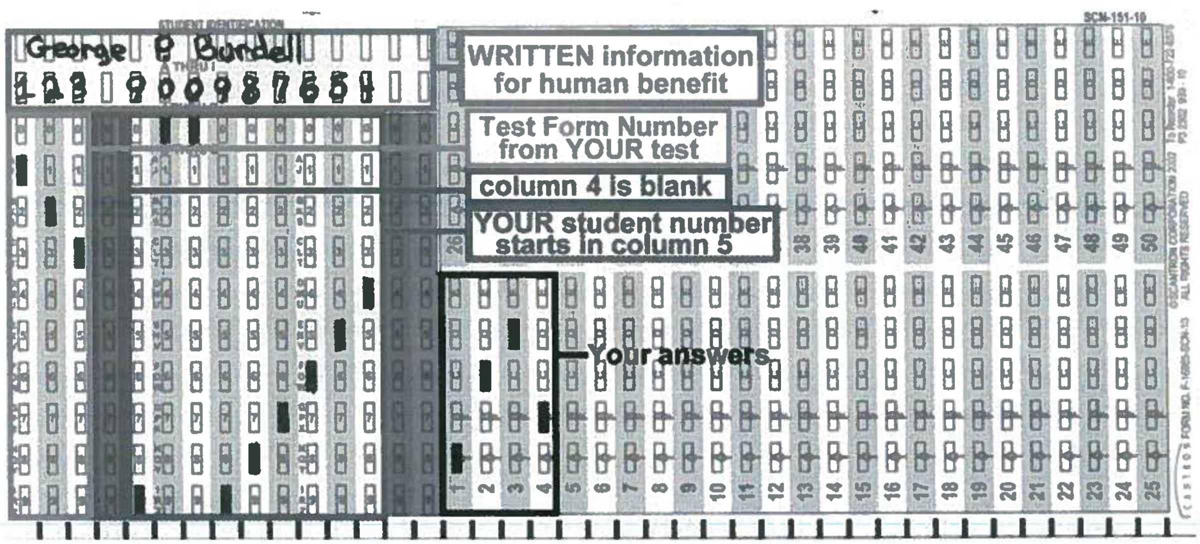
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

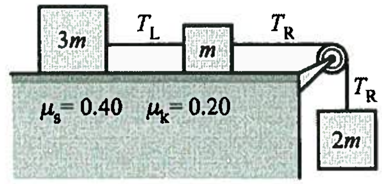
Your test form is: $\boxed{433}$



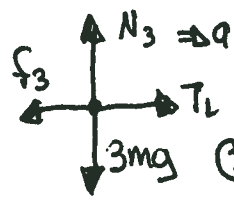
Our next test will be on Monday, April 6!

The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

[I] A block of mass $2m$ is suspended by an ideal cord that passes over a massless, frictionless pulley and is attached to a block of mass m lying on a rough horizontal surface. A second ideal cord attaches the second block to a third block, of mass $3m$. The friction coefficients between the two blocks and the surface are $\mu_s = 0.40$ and $\mu_k = 0.20$.



(A) *(10 points)* Static friction will not hold the blocks at rest; if released, they will slip along the surface. What will be the tensions in the two cords—T_L connecting m to $3m$ and T_R connecting $2m$ to m—after they are released? Express each tension as a multiple of mg.



$\Sigma \vec{F}_x = m\vec{a}_x$

$\langle +T_L\rangle + \langle -f_3\rangle = 3m\langle +a\rangle$

③ $\boxed{T_L - 3\mu_k mg = 3ma}$

$\Sigma \vec{F}_y = 0$

$\langle +N_1\rangle + \langle -mg\rangle = 0$

$N_1 = mg$

$f_1 = \mu_k mg$

$\Sigma \vec{F}_y = 0$

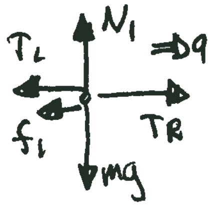
$\Rightarrow N_3 = 3mg$

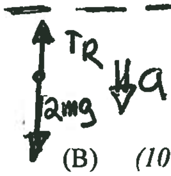
$f_3 = \mu_k 3mg$

$\Sigma \vec{F}_x = m\vec{a}_x$

$\langle +T_R\rangle + \langle -T_L\rangle + \langle -f_1\rangle = m\langle +a\rangle$

$\boxed{T_R - T_L - \mu_k mg = ma}$ ①





$\langle +T_R\rangle + \langle -2mg\rangle = 2m\langle -a\rangle$

$\boxed{2mg - T_R = 2ma}$ ②

① ② and ③ provide three equations in T_R, T_L, a

- Take ③ + ① − ②:

$2mg - 3\mu_k mg - \mu_k mg = 6ma$

$a = \frac{(2 - 4\mu_k)}{6} g = \boxed{0.2g}$

- Plug into ③

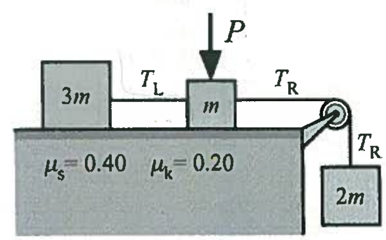
$T_L = 3ma + 3\mu_k mg = 0.6mg + 0.6mg$

$\boxed{T_L = 1.2mg}$

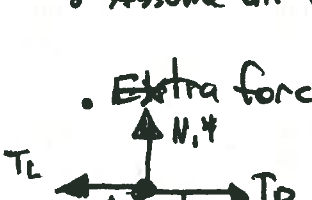
- plug into 2

$T_R = 2mg - 2ma = 2mg - 0.4mg = \boxed{1.6mg}$

(B) *(10 points)* By pushing straight down on block m, it is possible to hold all three blocks at rest, <u>without</u> slipping. What minimum downward push P will maintain the equilibrium of the blocks? Express your answer as a multiple (or fraction) of mg.



- Assume all accelerations are now zero
  $\Rightarrow$ diagram for $2m$ immediately gives $\boxed{T_R = 2mg}$
- Assume all friction forces are <u>maximal</u> static
  $\longrightarrow f_3 = \mu_s N_3$ and $f_1 = \mu_s N_1^*$
- ~~Extra~~ Extra force on m <u>changes</u> normal force $N_1 \rightarrow N_1^*$



$\langle +N_1^*\rangle + \langle -mg\rangle + \langle -P\rangle = 0 \rightarrow \boxed{N_1^* = mg + P} \rightarrow f_1 = \mu_s mg + \mu_s P$

$\Rightarrow$ Since m does not accelerate: $\langle +T_R\rangle + \langle -T_L\rangle + \langle -f_1\rangle = 0$

- <u>Diagram</u> for $3m$ does not change (other than $a \to 0$)

$\langle +T_L\rangle + \langle -f_3\rangle = 0 \rightarrow T_L = f_3 = \mu_s N_3 = \boxed{3\mu_s mg}$

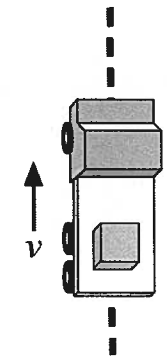
- Plug knowledge about T_L, T_R into horizontal equation for block m

$(2mg) - (3\mu_s mg) + (-\mu_s mg - \mu_s P) = 0$

$\mu_s P = 2mg - 4\mu_s mg \Rightarrow \boxed{P = \left(\frac{2}{\mu_s} - 4\right)mg = 1mg}$

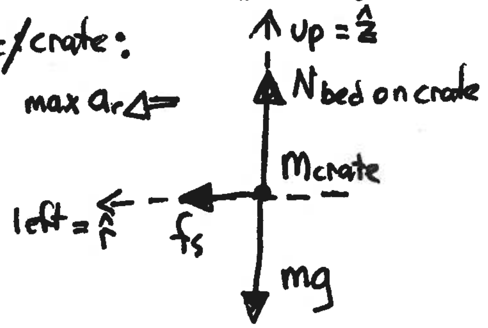
The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

[II] At right is a top-down view of a flatbed truck driving at constant speed v along a straight section of roadway. A crate lies on the truckbed, with coefficients of friction μ_k and μ_s between the crate and bed. The truck driver suddenly veers left (maintaining his speed v) to avoid a unicorn in the road ahead.



(15 points) Draw a free body diagram for the crate, *as seen from behind*, and determine the smallest possible turn radius the the truck can have without allowing the crate to slip off the truckbed. Express your answer in therms of v, μ_k, and/or μ_s, as well as g.

Rear view of truck/crate:



- static friction gives radial accel to crate
- min radius = max acceleration = max friction

$\Sigma \vec{F}_z = 0$

$\langle +N \rangle + \langle -mg \rangle = 0$

$N = mg$

$\Rightarrow f_s = \mu_s N = \mu_s mg \quad \rightarrow$ so: $\Sigma \vec{F}_r = m\vec{a}_r$

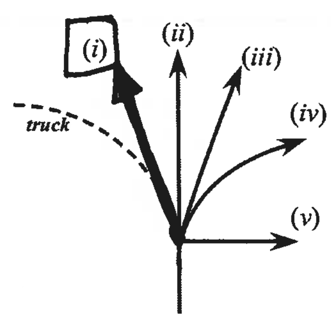
$\langle +f_s \rangle = m \langle +a_{max} \rangle = m \left\langle +\frac{v^2}{R_{min}} \right\rangle$

$\mu_s \cancel{m} g = \cancel{m} \frac{v^2}{R_{min}}$

$$\boxed{R_{min} = \frac{v^2}{\mu_s g}}$$

Question value 5 points — mark your answer in space #1 on the answer card.

(1) Suppose that the driver veers too sharply, and the crate slips off the bed. Which of the trajectories at right best depicts the path of the crate after it slides of the truckbed. (Assume the truck itself has continued its turn, following the dotted-line trajectory.)



(a) The crate follows path (*iv*).

* (b) The crate follows path (*ii*).

(c) The crate follows path (*v*).

(d) The crate follows path (*i*).

(e) The crate follows path (*iii*).

Note from free body diagram above:
while still on truck, crate feels a friction force to the left

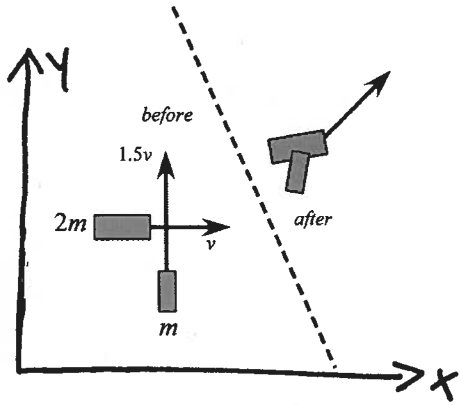
$\Rightarrow$ crate gains some leftward velocity as it slides off truck
so: $\vec{V}_{f,crate}$ is directed left of straight ahead

* [Path (ii) would be valid only if the truck bed were frictionless...]

The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[III] A UPS truck is travelling eastward through an intersection with speed v, when it is struck in the side by a station wagon travelling northward with a speed $1.5v$. As a result of the collision, the two vehicles become stuck, and skid together along the pavement.

(15 points) Assuming that the mass of the UPS truck is precisely twice that of the station wagon, what will be the velocity of the two vehicles immediately after the collision? Express your answer as a speed v_f that is a multiple (or fraction) of v, and a direction angle θ measured north of east.



initial momenta: truck $\vec{P}_{1i} = \langle +2mv \rangle \hat{i}$

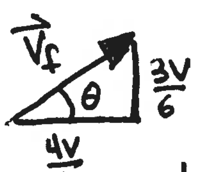
wagon $\vec{P}_{2i} = \langle +m \cdot \frac{3}{2}v \rangle \hat{j}$

$$\vec{P}_i = mv\left[2\hat{i} + \frac{3}{2}\hat{j}\right]$$

stick together: momentum is conserved, and $\vec{V}_{1f} = \vec{V}_{2f} = \vec{V}_f$ (no need for subscripts)

$$\vec{P}_f = (3m)\vec{V}_f \quad \Rightarrow \vec{P}_i = \vec{P}_f \quad \Rightarrow 3\cancel{m}\vec{V}_f = \cancel{m}v\left[2\hat{i} + \tfrac{3}{2}\hat{j}\right]$$

$$\vec{V}_f = \frac{v}{3}\left[2\hat{i} + \frac{3}{2}\hat{j}\right] = \langle \tfrac{4}{6}v \rangle \hat{i} + \langle \tfrac{3}{6}v \rangle \hat{j}$$ (getting common denominator)



magnitude: $|\vec{V}_f| = \sqrt{\frac{16}{36}v^2 + \frac{9}{36}v^2} = \sqrt{\frac{25}{36}v^2}$

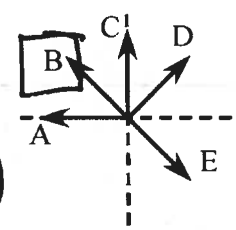
$$\boxed{V_f = \tfrac{5}{6}V}$$

$$\tan\theta = \frac{V_{fy}}{V_{fx}} = \frac{3V/6}{4V/6} \qquad \theta = \tan^{-1}\left(\tfrac{3}{4}\right) = \boxed{36.9°}$$

two-digit accuracy would probably be best here, so 37°

Question value 5 points — mark your answer in space #2 on the answer card.

(2) Which of the arrows in the diagram at right *best* characterizes the direction of the impulse delivered to the UPS truck by the station wagon?



(a) Direction B
(b) Direction D
(c) Direction A
(d) Direction E
(e) Direction C

① Clearly, truck has gained northward momentum (from car!)

② Clearly, car has gained eastward momentum (from truck!)

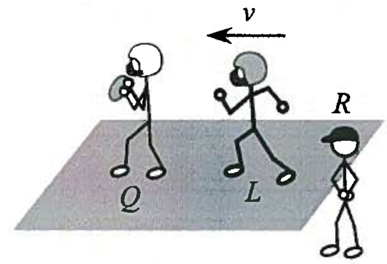
⇒ by 3rd Law, truck has lost eastward momentum

[it is also ~~clear~~ because V_{xf} for truck is less than V_o, from (A)]

① implies Northward impulse] to truck:
② implies westward impulse]

$\vec{J}_{\text{on truck}} =$

A football Quarterback is tackled from behind by a rushing defensive Lineman. The Quarterback (mass m) is stationary at the moment of impact, while the Lineman (mass $1.5m$) is moving with a speed v. Nearby, a stationary Referee observes the collision.



Question value 5 points

(3) Which of the three observers will see that the total momentum of the Quarterback and Lineman is a conserved quantity—as measured *in that observer's frame of reference*?

(a) For Q and L only.
(b) For all three observers.
(c) For R and Q only.
(d) For none of the three observers.
(e) For R only.

Q and L will both change their velocity, as a result of collision

⇒ Q and L are non-inertial observers (they experience an acceleration)

⇒ Q and L do NOT see Newton's Laws as being "true", from their POV

Since Conservation of Momentum is a result of 3rd Law, Q and L do not see their momentum as being conserved

Question value 5 points

(4) Assuming a coordinate system where L's initial velocity is in the negative direction, what is the vector impulse delivered to the L by Q, during the collision?

(a) $\vec{J} = \langle -2mv/3 \rangle$
(b) $\vec{J} = \langle -2mv/5 \rangle$
(c) $\vec{J} = \langle +mv/3 \rangle$
(d) $\vec{J} = \langle +3mv/5 \rangle$
(e) $\vec{J} = \langle -mv/2 \rangle$

① Determine $\vec{V}_f$ using totally inelastic collision:

$\vec{P}_i = \vec{P}_f$ (as seen by R!)

$$m\langle 0\rangle + \tfrac{3}{2}m\langle -V\rangle = (m+\tfrac{3}{2}m)\vec{V}_f = (\tfrac{5}{2}m)\vec{V}_f$$

$$\vec{V}_f = -\tfrac{3}{5}V$$

So: $\vec{J}_{toL} = \Delta\vec{P}_L = \vec{P}_{fL} - \vec{P}_{iL} = (\tfrac{3}{2}m)\langle -\tfrac{3}{5}V\rangle - (\tfrac{3}{2}m)\langle -V\rangle$

$= (\tfrac{3}{2}m)\langle -\tfrac{3}{5}V + V\rangle = (\tfrac{3}{2}m)\langle +\tfrac{2}{5}V\rangle$

$$\vec{J}_{toL} = \langle +\tfrac{3}{5}mV\rangle$$

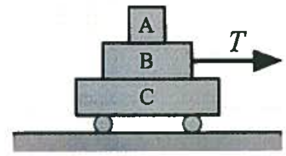
Question value 5 points

(5) Which statement below properly characterizes the forces acting on the players *during* the collision?

(a) Q experienced a greater magnitude of collision force, because L had a greater mass.
(b) L experienced a greater magnitude of collision force, because he experienced a larger velocity change.
(c) Q experienced a greater magnitude of collision force, because L was moving faster than Q at the moment of impact.
(d) The net force on Q and the net force on L are both zero because the total momentum of the system does not change.
(e) Both players experienced collision forces of equal magnitude.

3rd Law is absolute: the two collision forces, Q on L and L on Q are necessarily equal in magnitude

Block A rests on block B, which rests on block C, which in turn rests on frictionless rollers. The masses of the blocks are, in order, $m_A = M$, $m_B = 2M$, and $m_C = 4M$. The blocks are pulled together, by a cord attached to block B, having tension T.



Question value 5 points

(6) How many total Third Law force pairs are acting between the three blocks? (You may assume that mutual gravitational attractions *to each other* are negligible.)

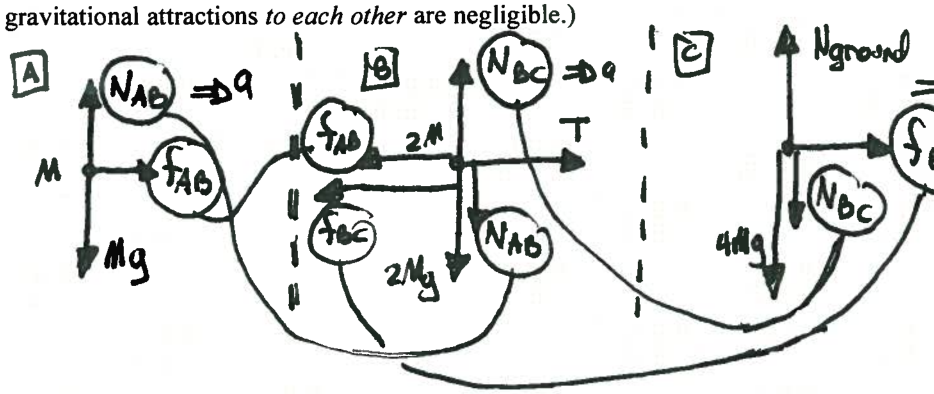
(a) two

(b) three

(c) five

(d) none

(e) four



• two friction forces (AB and BC)

• two normal forces (AB and BC)

Question value 5 points

(7) According to the Third Law, what force is paired with the upward normal force exerted by block C on block B?

(a) The weight of both blocks A and B, down on block C.

(b) The downward normal force by block B on block C.

(c) The weight of just block A, down on block B

(d) The downward normal force by block A on block B.

(e) The weight of just block B, downward.

→ pairs with a downward normal force

pairs with a force by B on C

ie — we are talking about the ~~two~~ "N_{BC}" vectors, in the diagrams above

Question value 5 points

(8) The coefficient of static friction between blocks A and B is $\mu_{AB} = 0.34$, and the coefficient of static friction between blocks B and C is $\mu_{BC} = 0.40$. What maximum acceleration can be imparted to all three blocks (by pulling the cord with some tension T_{max}), without anything slipping?

(a) $0.37g$

(b) $0.40g$

(c) $0.34g$

(d) $0.30g$

(e) $0.74g$

① • force f_{AB} accelerates block A : $\langle +f_{AB}\rangle = M\langle +a\rangle$

• vertical forces on A : $\langle +N_{AB}\rangle + \langle -Mg\rangle = 0 \rightarrow N_{AB} = Mg$

$\Rightarrow f_{AB,max} = \mu_{AB} N_{AB} = \mu_{AB} Mg$

So: max accel to A is $\cancel{M} a_{max} = \mu_{AB} \cancel{M} g = \mu_{AB} g = \boxed{0.34g}$ a_{max} for A

② Vertical forces on B:

$\langle +N_{BC}\rangle + \langle -N_{AB}\rangle + \langle -2Mg\rangle = 0$

$N_{BC} = 2Mg + N_{AB} = 3Mg$

This force determines max friction between B and C

$\boxed{f_{BC,max} = \mu_{BC} \cdot 3Mg}$

③ force f_{BC} accelerates block C:

$\langle +f_{BC}\rangle = (4M)\langle +a\rangle$

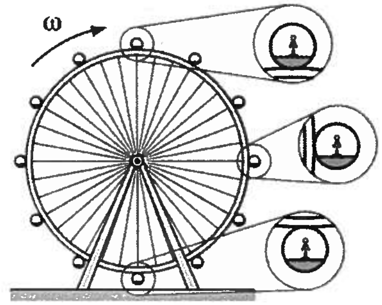
$\Rightarrow$ max accel when $f_{BC} = f_{BC,max} = 3\mu_{BC} Mg$

so that $a_{max} = \dfrac{3\mu Mg}{4M} = \dfrac{3}{4}\mu_{BC} g = \boxed{0.30g}$ a_{max} for C

④ choose the lesser value

$\boxed{a_{max}(\text{all}) = 0.30g}$

The London Eye is a huge Ferris wheel suspended overover the river Thames. Note that the passenger capsules on the Eye are designed so that the floor remains directly under one's feet at all times—the capsules do *not* "invert" as the Eye rotates. Assume the Eye has radius R and rotates at a fixed angular speed ω.



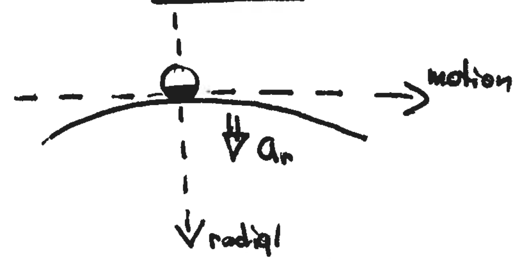
$$\sum \vec{F}_{radial} = m\vec{a}_{radial}$$

Question value 5 points

(9) What are the directions of: the net force on a passenger at the top of the loop, $(\sum \vec{F})_T$; and the net force on a passenger at the bottom of the loop, $(\sum \vec{F})_B$?

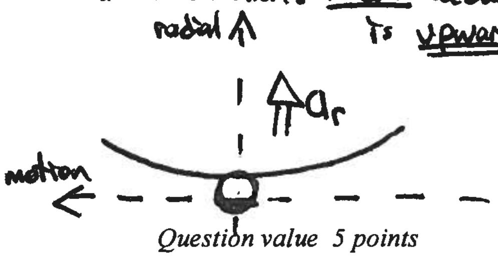
(a) $(\sum \vec{F})_T$ is ~~upward~~ and $(\sum \vec{F})_B$ is ~~downward~~.
(b) $(\sum \vec{F})_T$ is ~~upward~~ and $(\sum \vec{F})_B$ is upward.
(c) $\boxed{(\sum \vec{F})_T \text{ is downward and } (\sum \vec{F})_B \text{ is upward.}}$
(d) $(\sum \vec{F})_T$ and $(\sum \vec{F})_B$ are ~~both zero~~, so direction is irrelevant.
(e) $(\sum \vec{F})_T$ is downward and $(\sum \vec{F})_B$ ~~is downward~~.

at the top: radial acceleration is downward



so: $\boxed{\sum \vec{F} \text{ must be downward, at top (2nd Law)}}$

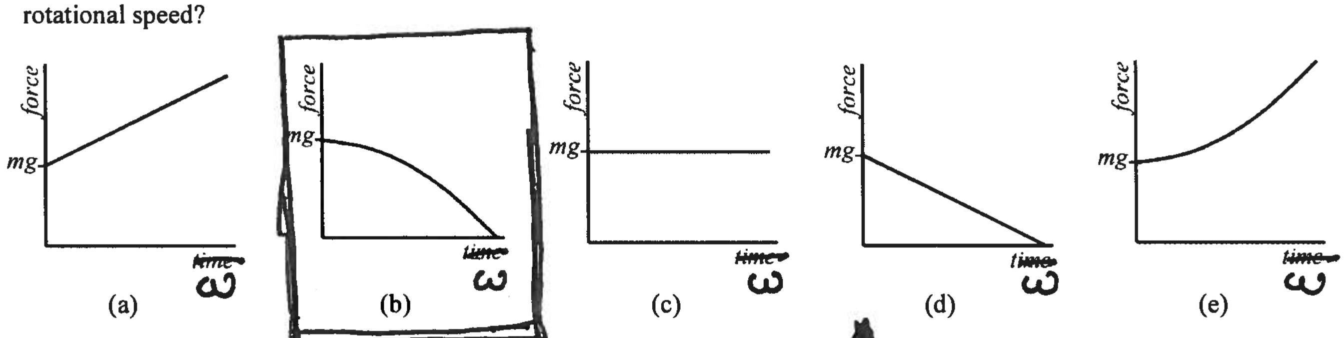
at the bottom: radial acceleration is upward



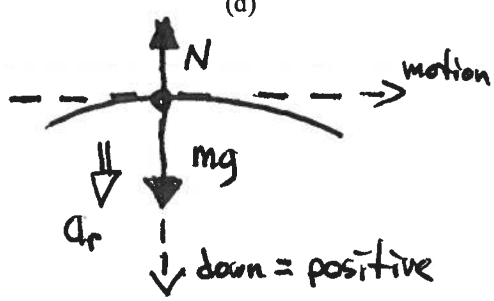
So: $\sum \vec{F}$ must be upward at bottom (2nd Law)

Question value 5 points

(10) Suppose that a deranged engineer sabotages the Eye so that its angular speed ω steadily increases. Which of the graphs below best characterizes the normal force by the floor on a passenger at the top of the loop, as a function of the Eye's rotational speed?



Free body diagram for passenger at top:



$$\langle +mg \rangle + \langle -N \rangle = m\langle +a_r \rangle$$

where radial acceleration is

$$a_r = \frac{(V_{Tangential})^2}{R} = \frac{(R\omega)^2}{R} = \omega^2 R$$

so: $mg - N = m\omega^2 R$

$-mg + N = -m\omega^2 R$

$$\boxed{N = m(g - \omega^2 R)}$$

- decreases as ω increases
- quadratic in ω: parabolic curve