Physics 2211M
Summer 2015
Test 3

Test form 533

Name Solutions

Recitation Section (see back of test):________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

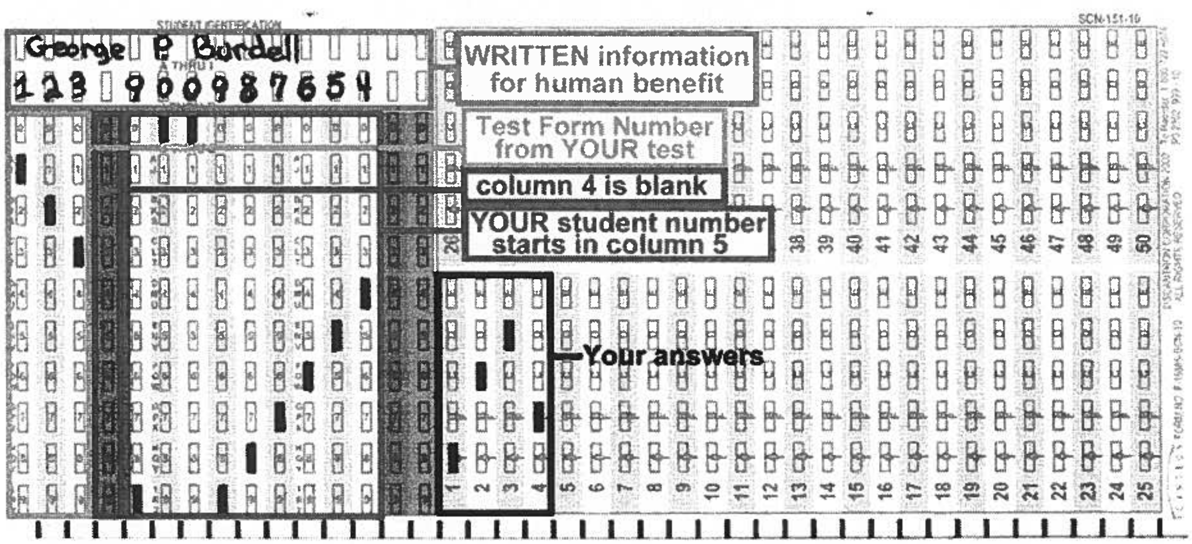
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test,** and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

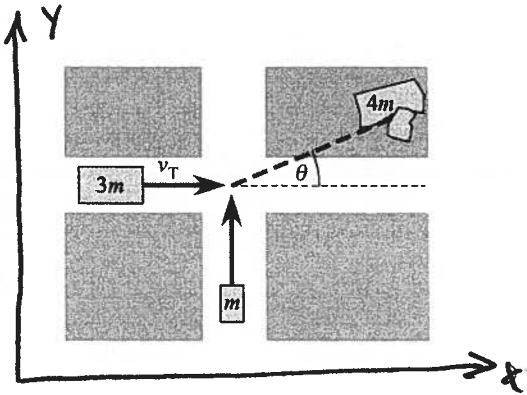
6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

Your test form is: 533



Our Final Exam will be on Wednesday, July 29!

The following problem will be hand-graded. <u>*Show all your work for this problem*</u>. *Make no marks and leave no space on your answer card for it.*



[I] *(20 points)* You are an accident investigator, working the scene of a collision between a sportscar (mass m) and a UPS truck (mass $3m$). The truck was initially moving due east into the intersection with a speed $v_T = 24$ mph (as confirmed by a GPS recorder on the truck). The car was moving due north into the intersection, and the driver claims that he was traveling below the posted speed of 25 mph. Skid marks from the collision indicate that the two vehicles stuck together, sliding in a direction $\theta = 22.5°$ north of east as they came to a stop.

Determine the actual speed of the sportscar just before the collision. Was the driver really obeying the speed limit, as he claimed?

$$\vec{P}_i = (3m)(+V_T\hat{i}) + (m)(+V_c\hat{j}) \quad \text{where } V_c = ??$$

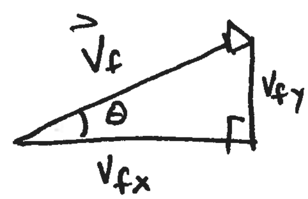
momentum is conserved $\vec{P}_i = \vec{P}_f$

where $\vec{P}_f = (4m)\vec{V}_f$, and $\vec{V}_f$ is directed at $\theta = 22.5°$ N of E

$$\vec{P}_f = \vec{P}_i$$

$$(4m)\vec{V}_f = 3mV_T\hat{i} + mV_c\hat{j}$$

$$\vec{V}_f = \underbrace{\tfrac{3}{4}V_T}_{V_{fx}}\hat{i} + \underbrace{\tfrac{1}{4}V_c}_{V_{fy}}\hat{j}$$



$$\tan\theta = \frac{V_{fy}}{V_{fx}} = \frac{\frac{1}{4}V_c}{\frac{3}{4}V_T} = \frac{V_c}{3V_T}$$

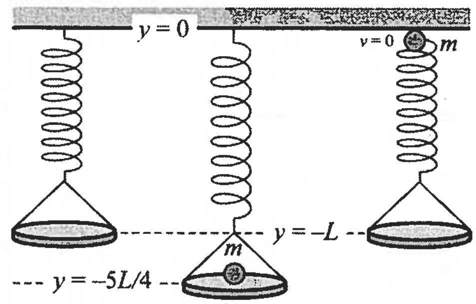
$$V_c = 3V_T\tan\theta$$
$$= 3V_T(0.414)$$

$$\boxed{V_c = 30\text{ mph}}$$

driver was <u>speeding</u> as he entered the intersection

The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

[II] *(20 points)* A pan of negligible mass hangs from a spring that is suspended from the ceiling. Initially, the pan hangs a distance L below the ceiling (*far left*). When a mass m is gently placed on the pan, the pan hangs at rest a distance $5L/4$ below the ceiling (*center*). If the same mass m is dropped from the ceiling onto the pan (*far right*), at what distance D below the ceiling will the pan momentarily come to a stop (*not shown*)? *Express your answer in terms of the parameter L.*



① Equilibrium of spring + mass:

$F_s = -k\Delta y$

$F_g = mg$

• note that $\Delta y = y - y_{eq} = (-\tfrac{5}{4}L) - (-L) = -\frac{L}{4}$

[negative Δy is consistent with $\vec{F}_s = -k\vec{\Delta y}$ being upward]

require: $\boxed{\Sigma \vec{F}_y = 0}$

$\langle -k(-\tfrac{L}{4})\rangle + \langle -mg\rangle = 0 \rightarrow \frac{kL}{4} = mg \quad \boxed{k = \frac{4mg}{L}}$

② Conservation of energy when mass is dropped from ceiling ($y=0$):

$y = 0 \qquad U_{gi} = 0 \quad U_{si} = 0 \quad K_i = 0 \rightarrow E_i = 0$

$y_{eq} = -L$

$\vec{\Delta y}_{spring} = \langle -(D-L)\rangle$

$y_f = -D \qquad U_{gf} = -mgD \quad U_{si} = \frac{1}{2}k\underbrace{(D-L)}_{|\Delta y_{spring}|}^2 \quad K_f = 0$

$E_i = E_f \rightarrow 0 = (-mgD) + \frac{1}{2}k(D^2 - 2DL + L^2)$

but $\frac{1}{2}k = \frac{1}{2}\left(\frac{4mg}{L}\right) = \frac{2mg}{L}$

$0 = -mgD + \frac{2mg}{L}(D^2 - 2DL + L^2)$

$0 = -DL + 2D^2 - 4DL + 2L^2 = 2D^2 - 5DL + 2L^2$

quadratic: $D = \frac{-(-5L) \pm \sqrt{25L^2 - 4(2)(2L^2)}}{2(2)} = \frac{5L \pm \sqrt{9L^2}}{4} = \frac{5L \pm 3L}{4}$

since D must be $> L$, choose positive root:

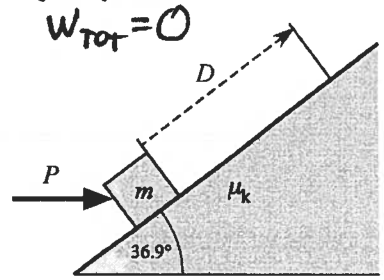
$D = \frac{5L + 3L}{4} = \frac{8L}{4} \Rightarrow \boxed{D = 2L}$

The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

$K = \text{constant}: \Delta K = 0$

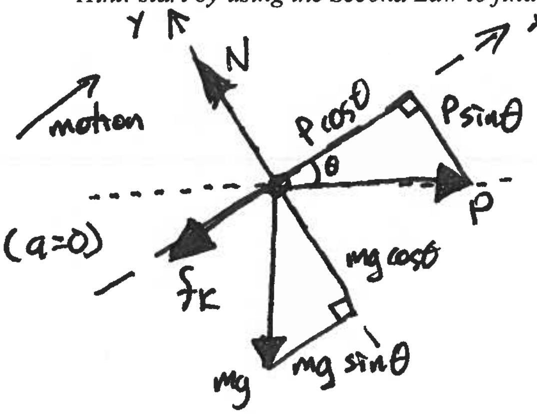
$W_{TOT} = 0$

[III] *(20 points)* A block of mass m is pushed up a rough ramp at constant speed by the application of a horizontal pushing force P. The ramp ~~is inclined~~ at an angle $\theta = 36.9°$ above the horizontal, and the coefficient of friction for the ramp is $\mu_k = 0.500$.



Identify all of the forces acting on the block, and calculate the work done by each force, as the block moves a distance D up the ramp. **Express each answer in terms of M, g, and/or D.**

Hint: start by using the Second Law to find each force as some multiple of mg!



$\sum \vec{F}_y = 0$

$\langle +N \rangle + \langle -P\sin\theta \rangle + \langle -mg\cos\theta \rangle = 0$

(A) $\boxed{N - \frac{3}{5}P = \frac{4}{5}mg}$ (using 345 geometry)

$\sum \vec{F}_x = 0$

$\langle +P\cos\theta \rangle + \langle -f_K \rangle + \langle -mg\sin\theta \rangle = 0$

(B) $\boxed{\frac{4}{5}P - \frac{1}{2}N = \frac{3}{5}mg}$ (using geometry and $f_K = \mu_k N = \frac{1}{2}N$)

geometry of 3-4-5 triangle:

$\cos 36.9° = \frac{4}{5}$

$\sin 36.9° = \frac{3}{5}$

(A) and (B) give us two equations in P, N

take 2B+A: $\left(\frac{8}{5}P - N\right) + \left(N - \frac{3}{5}P\right) = \left(\frac{6}{5}mg\right) + \left(\frac{4}{5}mg\right)$

$\rightarrow \boxed{P = 2mg}$

given P, N is found from:

$N = \frac{3}{5}P + \frac{4}{5}Mg = \frac{6}{5}mg + \frac{4}{5}mg = \frac{10}{5}mg \rightarrow \boxed{N = 2mg} \rightarrow \boxed{f_K = \frac{1}{2}N = mg}$

Now find work by each, during displacement $\vec{\Delta x} = \langle +D \rangle \hat{i}$

- $\vec{N} \perp$ to $\vec{\Delta x} \rightarrow \boxed{W_N = 0}$ yes — you do have to point this out!
- $\vec{f}_K$ is opposite to $\vec{\Delta x} \rightarrow \boxed{W_f = -f_K D = -mgD}$
- for $\vec{P}$, use component $\vec{P}_x = \langle +P\cos\theta \rangle = +\frac{4}{5}P \rightarrow W_p = P_x \Delta x = \frac{4}{5}PD = \frac{4}{5}(2mg)D$ $\boxed{W_p = +\frac{8}{5}mgD}$
- for $\vec{F}_g$, note $\vec{F}_{g,x} = \langle -mg\sin\theta \rangle = -\frac{3}{5}mg$

$W_g = F_{g,x}\Delta X = \left(-\frac{3}{5}mg\right)(D) \rightarrow \boxed{W_g = -\frac{3}{5}mgD}$

(note that all four answers sum to zero: $W_{TOT} = \Delta K = 0$)

Question value 4 points

(1) A hammer is dropped from the roof of a tall building. Ignoring drag, the power delivery <u>by gravity</u> to the hammer will be:

(a) constant and positive throughout the hammer's descent.
$\boxed{\text{(b) initially zero, but increasing uniformly to greater and greater positive values as the hammer descends.}}$
(c) initially zero, but decreasing uniformly to greater and greater negative values as the hammer descends.
(d) zero, because no power is ever delivered by a conservative force like gravity.
(e) constant and negative throughout the hammer's descent.

instantaneous power delivery by a force is $P = \vec{F} \cdot \vec{v}$

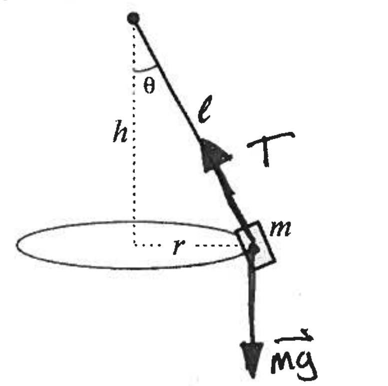
for hammer in free fall, $\vec{F} = \langle -mg \rangle \hat{j} = \text{constant}$

$\vec{v} = \langle -gt \rangle \hat{j} \rightarrow$ increases linearly from zero

$\Rightarrow \vec{P} = +mg \cdot gt = mg^2t \rightarrow$ $\boxed{\text{increases linearly, to larger positive values, starting from } P=0 \text{ at } t=0}$

Question value 8 points

(2) A block hanging from a cord is whirled in a horizontal circle, forming a conical pendulum. What forces act, and what work do they do? (Assume the block is in uniform circular motion.)



(a) Tension *along* the cord and gravity straight down. Gravity does positive work while tension does an equal amount of negative work.

(b) Tension *along* the cord and gravity straight down. Gravity does negative work while tension does an equal amount of positive work.

(c) Tension, gravity, and the force of the circular motion. Tension and gravity do zero work, while the force of the motion does positive work.

$\boxed{\text{(d) Tension } \textit{along} \text{ the cord and gravity straight down. Neither force does } \textit{any} \text{ work.}}$

(e) Tension, gravity, and the centripetal force. Since they are all perpendicular to the motion, they all do zero work.

note:
$v = \text{constant}$
$\rightarrow K = \text{constant}$
$\rightarrow \Delta K = 0$
$\rightarrow W_{TOT} = 0$

$W = \vec{F} \cdot \Delta \vec{s} \longrightarrow \int \vec{F} \cdot d\vec{s}$

during a <u>short</u> time interval dt, let work be dW:

$dW = \vec{F} \cdot d\vec{s} = \vec{F} \cdot (\vec{v}\,dt)$

↳ small displacement

① gravity is vertically <u>down</u>, displacement $d\vec{s}$ is horizontal, tangent to circle

$\vec{F}_g \perp d\vec{s}$ so $dW = 0$ at all times $\rightarrow \boxed{W_{grav} = 0}$

② Tension is directed <u>up</u> and <u>radially in</u>

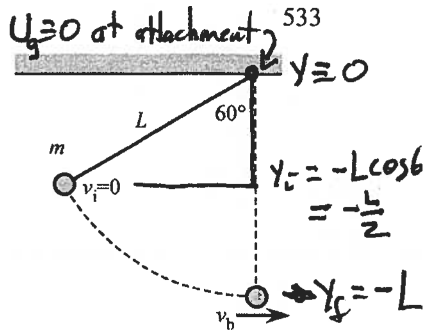
↳ "radial" is $\perp$ to tangential $d\vec{s}$

↳ "up" is $\perp$ to tangential $d\vec{s}$

$\rightarrow \boxed{W_T = 0}$ because $\vec{T} \perp d\vec{s}$

Question value 8 points

(3) In the figure at right, a child of mass m is riding a rope-swing of length L. If the child swings back and forth making a maximum angle of 60° from the vertical, what will be the tension in the rope at the very bottom of her (circular!) arc?



(a) $mg/2$

(b) $\boxed{2mg}$

(c) $(3/2)\,mg$

(d) mg

(e) $(\sqrt{3}/2)\,mg$

① Conservation of energy: $i \to b$

$K_i + U_{g\,i} = K_b + U_{g\,b}$

$0 + (-mg\frac{L}{2}) = \frac{1}{2}mv_b^2 + (-mgL)$

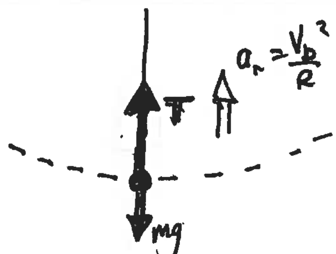
$\frac{1}{2}mv_b^2 = +\frac{mgL}{2} \to v_b^2 = gL$

$U_{gi} = mg\,y_i$
$= mg(-\frac{L}{2})$

$U_{gf} = mg\,y_f$
$= mg(-L)$

② At $\underline{\underline{b}}$, child is in circular motion

$\to$ radial acceleration $a_r = \frac{v_b^2}{R}$ toward top of rope



$a_r = \frac{v_b^2}{R} = \frac{v_b^2}{L} = \frac{gL}{L} = g$

$\sum \vec{F}_r = m\vec{a}_r$ for circular motion

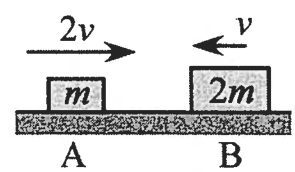
$\langle +T \rangle + \langle -mg \rangle = m\langle +a_r \rangle = m\langle +g \rangle$

$\Rightarrow \boxed{T = 2mg}$

(From diagram, it should be clear that T <u>must</u> <u>be</u> <u>greater</u> <u>than</u> mg)

Question value 8 points

(4) The diagram at right displays a head-on collision between two blocks of unequal mass. What are the two blocks' final velocities, if the collision is <u>perfectly elastic</u>?



$\vec{V}_{Ai} = \langle +2v \rangle$ $\vec{V}_{Bi} = \langle -v \rangle$

① Collision: $\vec{P}_i = \vec{P}_f$

② special case: perfectly elastic

$\to$ we <u>also</u> know $K_i = K_f$

(a) $\vec{v}_A = \langle -v \rangle$ and $\vec{v}_B = \langle -v/2 \rangle$

(b) $\vec{v}_A = \langle +v/2 \rangle$ and $\vec{v}_B = \langle +v \rangle$

(c) $\vec{v}_A = \langle -v \rangle$ and $\vec{v}_B = \langle +v/2 \rangle$

(d) $\boxed{\vec{v}_A = \langle -2v \rangle \text{ and } \vec{v}_B = \langle +v \rangle}$

(e) $\vec{v}_A = \langle -2v \rangle$ and $\vec{v}_B = \langle -v \rangle$

① Note: $\vec{P}_i = m\langle +2v \rangle + 2m\langle -v \rangle = 0$

$\to$ <u>only</u> answers with $\vec{P}_f = 0$ are possible

(so $\vec{V}_{Af}$ and $\vec{V}_{Bf}$ <u>must</u> have <u>opposite</u> signs!)

$\to$ only (c) and (d) satisfy this condition, with $\vec{P}_f = 0$

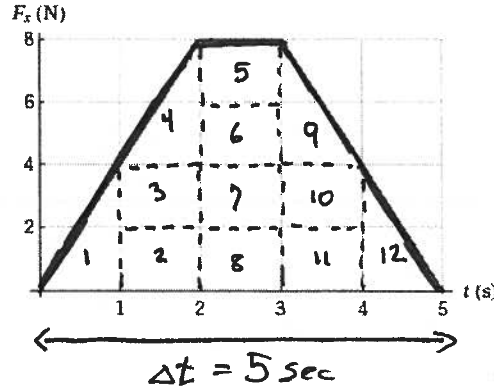
② $K_i = \frac{1}{2}(m)(2v)^2 + \frac{1}{2}(2m)v^2 = 2mv^2 + mv^2 = 3mv^2$

$\to$ only (d) and (e) have $K_f = 3mv^2$

only case (d) above satisfies <u>both</u> conservation conditions

Question value 8 points

(5) The graph at right displays the horizontal force on a tennis ball by a wall, during the time interval that the ball is rebounding off the wall. What is the magntitude of the average force exerted on the ball by the wall?



- $\boxed{\text{(a) 4.8 N}}$
- (b) 4.0 N
- (c) 1.6 N
- (d) 8.0 N
- (e) 2.4 N

① Impulse $\vec{J} = \int \vec{F} dt$

= area under curve

From figure, this area is "12 boxes"

② Each box has "height" 2 N, "width" 1 sec

→ Area of 1 box is 2 N·s → "A_{TOT}" = $\vec{J}$ = +24 N·s

③ Average Force: the "constant" force that delivers the same impulse in the same time!

$$\int \vec{F} dt = \vec{J} = \vec{F}_{av} \Delta t \qquad \boxed{|\vec{F}_{av}| = \frac{|\vec{J}|}{\Delta t} = \frac{24 \text{ N·s}}{5 \text{ s}} = 4.8 \text{ N}}$$

Question value 4 points

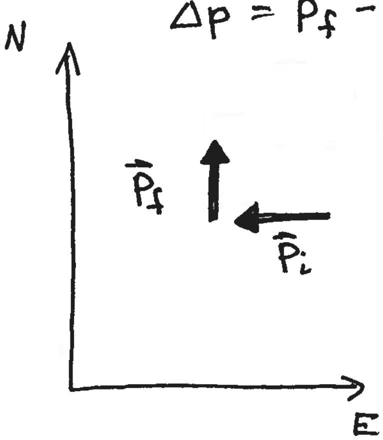
(6) A car is traveling due west at some speed v_o. It then turns right, and ends up traveling due north with the same speed v_o. What impulse (if any) does the ground deliver to the car during this process?

- (a) ~~There is no impulse at all during this process, because the car's speed didn't change.~~ Vectors don't work that way...
- $\boxed{\text{(b) There is a northeastward-directed impulse on the car by the ground.}}$
- (c) There is a ~~south~~eastward-directed impulse on the car by the ground.
- (d) There is a north~~westward~~-directed impulse on the car by the ground.
- (e) There is a ~~southwestward~~-directed impulse on the car by the ground.

impulse-momentum theorem: $\vec{J} = \Delta\vec{p}$ is a **VECTOR** relationship!

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = \vec{p}_f + (-\vec{p}_i)$$

→ graphically:



$\boxed{\vec{J} \text{ is north-eastward}}$

eastward impulse to cancel westward momentum

– plus –

northward impulse to add northward momentum