

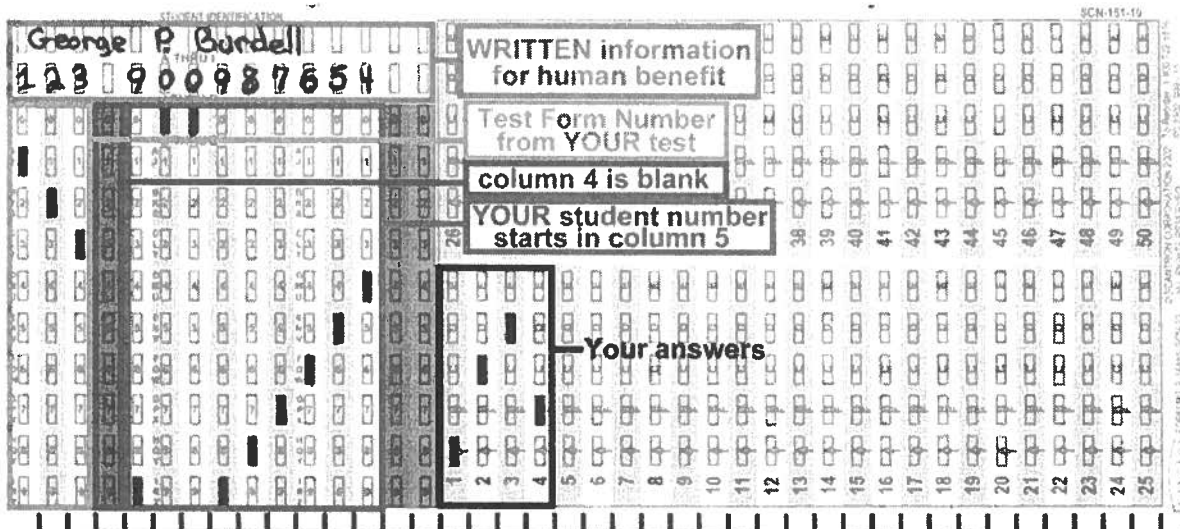
**Test 2**

Recitation Section (see back of test): \_\_\_\_\_

- 1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".
- 2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.
- 3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.
- 4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**
- 5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been graded. **Quiz grades become final when the next quiz is given.**
- 6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**



Your test form is: **251**

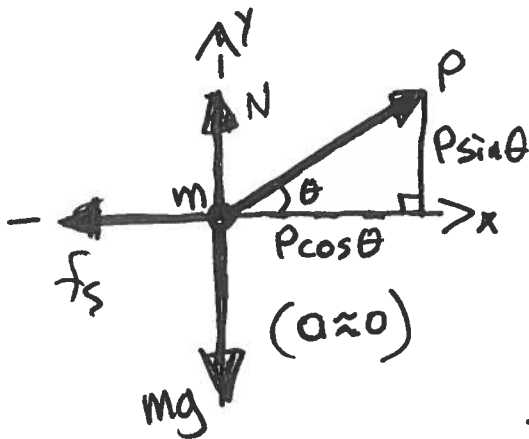
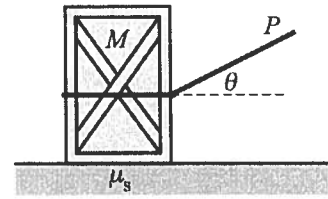


**Our next test will be on Wednesday, July 20!**

The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

- III (20 points) A large crate of mass  $M$  rests on a rough floor having coefficient of static friction  $\mu_s = 0.64$ . The crate is pulled by a cord that is angled at  $\theta = 36.9^\circ$  above the horizontal. With what *minimum* force must the crate be pulled in order for it to actually begin slipping along the surface?

Draw a free body diagram as part of your solution. Its clarity and quality will be graded as part of the problem!



"Minimum force to start slipping" is synonymous with

"Maximum force without slipping"

(any extra, teeny-tiny bit of force will push past the limit...)

$$\rightarrow \text{we can invoke } f_s = f_{s,\max} = \mu_s N$$

$$\textcircled{1} \quad \Sigma \vec{F}_y = 0 \rightarrow \langle +N \rangle + \langle +P \sin \theta \rangle + \langle -mg \rangle = 0$$

$$N = mg - P \sin \theta$$

$$\text{so } f_s = \mu_s N = \mu_s mg - \mu_s P \sin \theta$$

$$\textcircled{2} \quad \Sigma \vec{F}_x = 0$$

$$\rightarrow \langle +P \cos \theta \rangle + \langle -f_s \rangle = 0$$

$$P \cos \theta - [\mu_s mg - \mu_s P \sin \theta] = 0$$

$$P [\cos \theta + \mu_s \sin \theta] = \mu_s mg$$

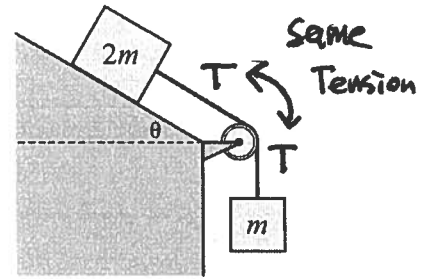
$$P = \frac{\mu_s}{\cos \theta + \mu_s \sin \theta} mg = \frac{0.64}{0.8 + (0.64)(0.6)} mg$$

$$P = 0.54 mg$$

[P is a multiple of mg, and thus has obvious units of force]

The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

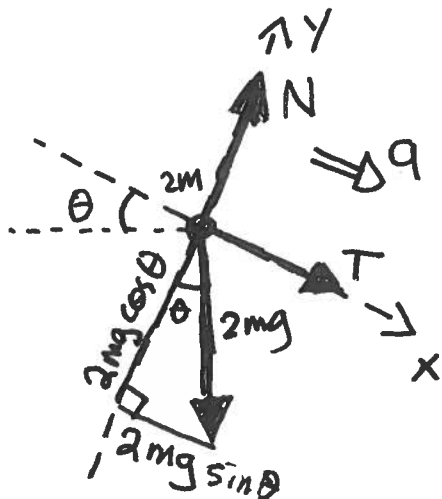
- III (20 points) A large block of mass  $2m$  is initially held on a frictionless downsloping surface inclined at an angle  $\theta = 30^\circ$  below the horizontal. An ideal cord is attached to the block, passing over a massless and frictionless pulley to where the other end is attached to a smaller block of mass  $m$  that hangs freely in space.



Find the tension in the cord and the acceleration of the two blocks, after the larger block is released.

Draw a free body diagram as part of your solution. Its clarity and quality will be graded as part of the problem!

Note well - when released, both blocks will begin accelerating



$$\sum \vec{F}_y = 0 \rightarrow \langle +N \rangle + \langle -2mg \cos \theta \rangle = 0$$

(not really needed here)

$$\sum \vec{F}_x = m \vec{a}_x$$

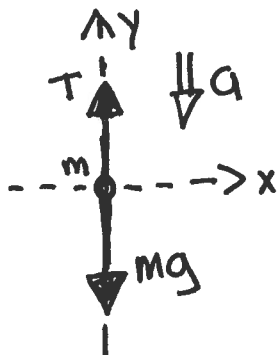
$$\langle +T \rangle + \langle +2mg \sin \theta \rangle = (2m) \langle +a \rangle$$

big block!

$$T + 2mg \sin \theta = 2ma$$

$$T + 2mg \left(\frac{1}{2}\right) = 2ma$$

$$\text{so } \boxed{T + mg = 2ma} \quad (1)$$



Note well: if up=pos, then  $\vec{a}_y$  is negative!  
 $\vec{a}_y = \langle -a \rangle$

$$\sum \vec{F}_y = m \vec{a}_y$$

$$\langle +T \rangle + \langle -mg \rangle = m \langle -a \rangle$$

$$\boxed{T - mg = -ma} \quad (2)$$

Two equations in T, a:

add (1) + 2x(2) to eliminate a:

$$(T + mg) + 2(T - mg) = 2ma + 2(-ma)$$

$$3T - mg = 0$$

$$\boxed{T = \frac{mg}{3}}$$

subtract (1) - (2) to eliminate T:

$$(T + mg) - (T - mg) = 2ma - (-ma)$$

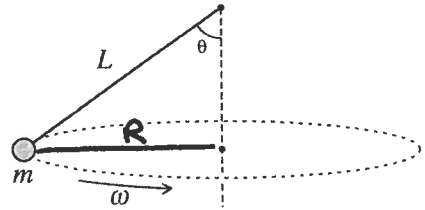
$$2mg = 3ma \rightarrow \boxed{a = \frac{2}{3}g}$$

T obviously has units of Force

a obviously has units of accel

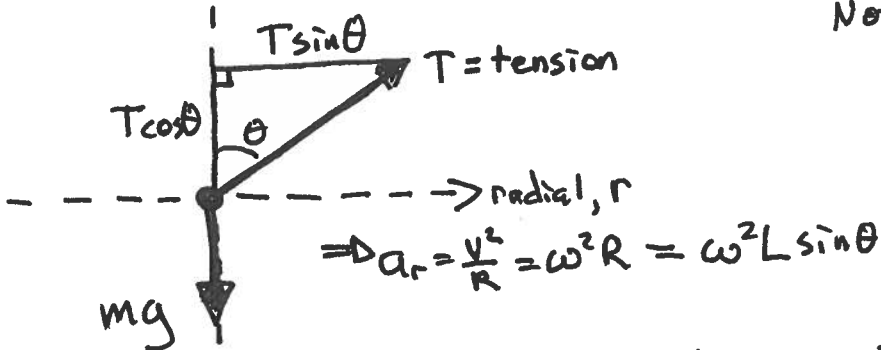
The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

- [III] (20 points) A conical pendulum consists of a mass  $m$  suspended from a cord of length  $L$ . The mass is whirled in a horizontal circle such that the cord traces out a cone that is oriented at an angle  $\theta$  from the vertical. Determine the required angular speed  $\omega$  for the conical pendulum that will result in a cone angle of  $\theta = 60^\circ$ . Express your answer in terms of the symbols  $L$  and  $g$ .



Draw a free body diagram as part of your solution. Its clarity and quality will be graded as part of the problem!

↑ Vertical,  $z$



Note: from geometry,  
 $R = L \sin \theta$

① Require  $\sum \vec{F}_z = 0 \rightarrow \langle +T \cos \theta \rangle + \langle -mg \rangle = 0$

$$\boxed{T = \frac{mg}{\cos \theta}}$$

② Require  $\sum \vec{F}_r = m \vec{a}_r \rightarrow \langle +T \sin \theta \rangle = m \langle +\omega^2 L \sin \theta \rangle$

$$T \sin \theta = m \omega^2 L \sin \theta$$

$$\boxed{T = m \omega^2 L}$$

So,  $\frac{mg}{\cos \theta} = m \omega^2 L$

$$\omega^2 = \frac{g}{L \cos \theta} = \frac{g}{L \cdot \frac{1}{2}} = \frac{2g}{L}$$

$$\boxed{\omega = \sqrt{\frac{2g}{L}}}$$

Note that  $g \Rightarrow [\text{accel}] = [\text{length units} / (\text{time units})^2]$

$L \Rightarrow [\text{length units}]$

$$\frac{g}{L} \Rightarrow \left[ \frac{\text{length} / \text{time}^2}{\text{length}} \right] = \left[ \frac{1}{\text{time}^2} \right]$$

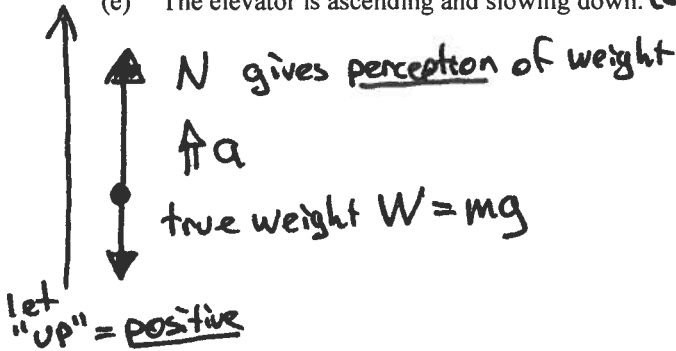
So  $\omega$  has units  $\left[ \frac{1}{\text{time}} \right] \rightarrow$  appropriate for "radians/second"

Question value 8 points

- (1) A child having *true* (or *gravitational*) weight  $W$  is riding in an elevator. At a particular moment, the child's *perceived* (or *apparent*) weight is greater than  $W$ . Which of the following statements accurately describes the possible motion of the elevator?

- (a) The elevator is descending and slowing down.
- (b) The elevator is in free fall. (accel = yes)
- (c) This cannot be a real situation, because apparent weight is always the same as gravitational weight.
- (d) The elevator is ascending at a constant speed.
- (e) The elevator is ascending and slowing down. (accel = yes)

Apparent weight will NOT be equal to true gravitational force whenever reference frame is accelerating



$$\sum F_y = m\vec{a}_y$$

$$\langle +N \rangle + \langle -W \rangle = m\vec{a}_y$$

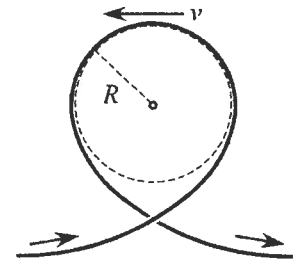
$N > W$  implies  $\vec{a}_y = \text{positive/upward}$

true when:

- moving up and gaining speed
- or • moving down and losing speed

Question value 8 points

- (2) As a safety engineer, you are reviewing the design for a new rollercoaster ride at Six Flags. The centerpiece of the design is a huge double-loop, in which the cars pass twice through an inverted loop of radius  $R$  while travelling at a speed  $v$ . When you "do the math" you see that the ride is unsafe, with passengers tending to fall out at the top of the loop. What changes to the loop might make the ride safe?



- (a) Increase the cars' speed, and/or decrease the loop radius.
- (b) Decrease the cars' speed, and/or decrease the loop radius.
- (c) Just strap the passengers in more tightly and hope for the best.
- (d) Decrease the cars' speed, and/or increase the loop radius.
- (e) Increase the cars' speed, and/or increase the loop radius.

Unsafe: passengers "fallout" because pull of gravity provides too much force - more than is actually needed to stay on circular path:  $g > a_r$

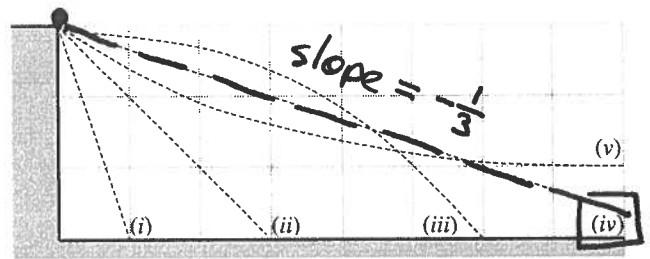
Safe:  $a_r > g$ : extra force (downward push by track) is needed to stay on circular path

So - to make ride safe, make  $a_r$  larger

Since  $a_r = \frac{v^2}{R} \rightarrow$  make  $v$  larger and/or make  $R$  smaller

Question value 8 points

- (3) A toy rocket of mass  $m$  is placed on its side at the edge of a cliff, with its nose oriented horizontally. The rocket motor ignites, generating a thrust that is three times the weight of the rocket. After it leaves the cliff, tiny gyroscopes keep the rocket in a horizontal orientation. Which of the plots at right best depicts the trajectory of the rocket?



- (a) Plot (ii).
- (b) Plot (iv).**
- (c) Plot (v).
- (d) Plot (iii).
- (e) Plot (i).

Starting at rest :  $\vec{v}_{xi} = 0 = \vec{v}_{yi}$   
 Assume toy starts at origin  $\vec{x}_i = 0 = \vec{y}_i$

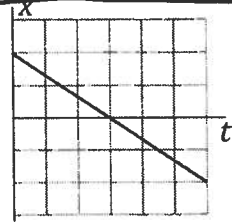
•  $\vec{x} = \vec{x}_i + \vec{v}_{ix} \Delta t + \frac{1}{2} \vec{a}_x \Delta t^2$  horizontal position  
 $x = \frac{3}{2} g \Delta t^2$

•  $\vec{y} = \vec{y}_i + \vec{v}_{iy} \Delta t + \frac{1}{2} \vec{a}_y \Delta t^2$  vertical position  
 $y = -\frac{1}{2} g \Delta t^2$

comparing:  
 $y = -\frac{1}{3} x$   
 path is a straight line, with shallow, negative slope

Question value 8 points

- (4) The figure at right shows the position-versus-time graph for an object travelling along the  $x$ -axis. Which of the statements below *best* characterizes the net force that is acting on the object?



- (a) A constant, negatively directed net force is acting on the object.
- (b) No net force is acting on the object.**
- (c) No forces at all are acting on the object.
- (d) A constant, positively-directed net force is acting on the object.
- (e) There is a net force that is initially positively directed, but then drops to zero and then becomes negatively-directed.

Constant velocity  
 $\Rightarrow$  object is in uniform motion

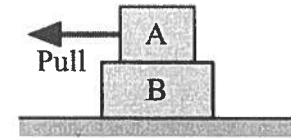
1<sup>st</sup> Law: uniform motion is natural, and requires no cause

ie **no net force on object**

$x$ -vs- $t$  is linear  
 $v_x$ -vs- $t$  is constant  
 $a_x = 0$   
 by 2<sup>nd</sup> Law,  $\sum \vec{F}_x = m\vec{a}_x = 0$

The next two questions involve the following situation:

Block A rests atop block B, which rests on rough ground. Block A is pulled to the left at constant speed, in such a way that block B moves with block A.



B moves with A, so: **B moves at constant speed!**

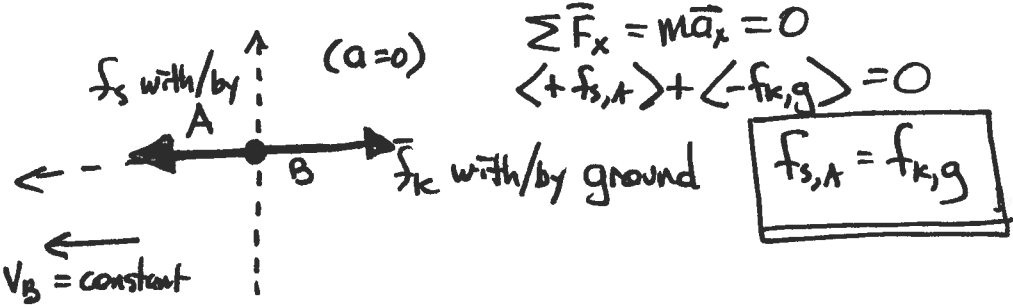
↳ **zero acceleration** → **zero net force**

Question value 4 points

(5) What are the horizontal forces acting on block B?

- (a) A leftward friction force from Block A, and a rightward friction force from the ground—with both forces being equal in magnitude.
- (b) A leftward friction force from Block A, only.
- (c) A leftward friction force from Block A, and a rightward friction force from the ground—with the leftward force being greater than the rightward force.
- (d) A rightward friction force from Block A, and a leftward friction force from the ground—with both forces being equal in magnitude.
- (e) A rightward friction force from Block A, only.

- B does move left, along rough ground  
→ Kinetic friction force by ground, **TO THE RIGHT**
- B does not slip out from under A: block A pulls B with it  
→ static friction on B by A, **TO THE LEFT**

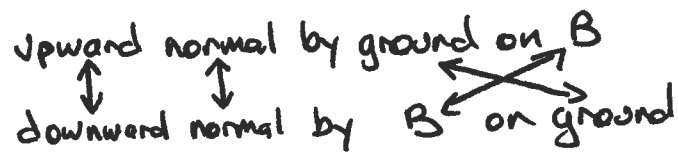


Question value 4 points

(6) What force (if any) is paired with the upward normal force by the ground on block B?

- (a) The weight of both blocks A and B, pushing down against the ground.
- (b) The weight of block B, pushing down against the ground.
- (c) The normal force by the bottom of block B, pushing down on the ground.
- (d) The normal force by block A, pushing down against block B.
- (e) The gravitational force by block B, pulling upward on the entire earth.

[should also say, "According to 3rd Law..."]



Also note: only answer (c) describes a force whose magnitude, under any and all circumstances, must necessarily be equal to the magnitude of the normal force on B by the ground