Physics 2211 ABC | Test form **411** | Name Official Solution Set

Fall 2014

Test 1

Recitation Section (see back of test):__________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

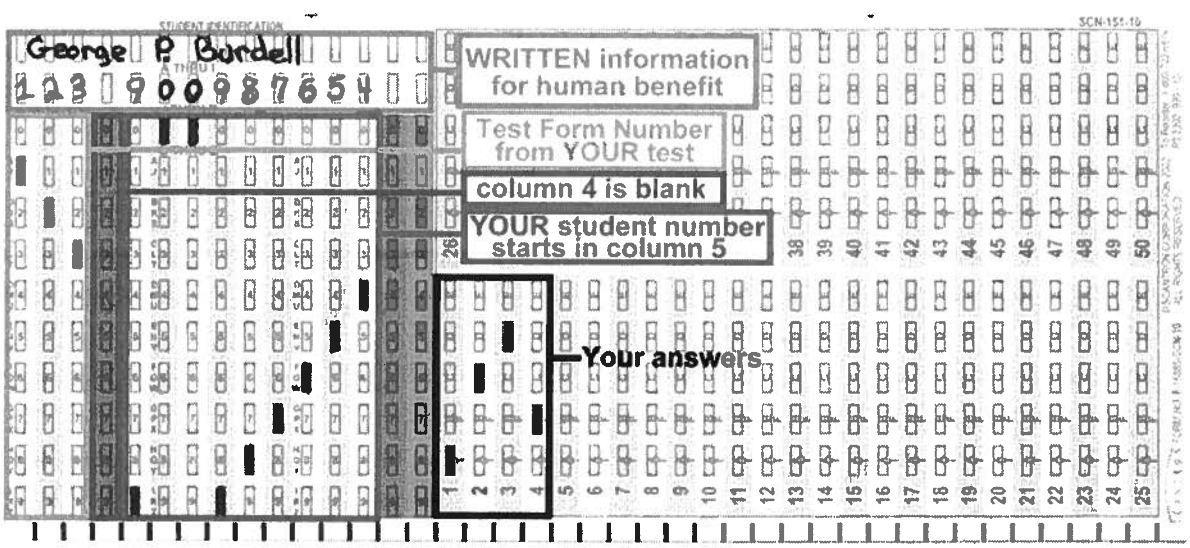
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

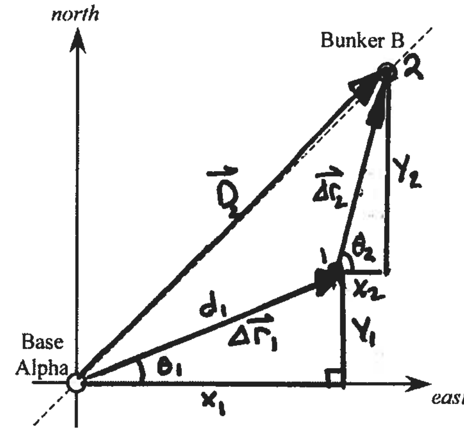
6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

Your test form is: 411



The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[I] You are a settler in the first permanent human colony on Mars. You set out from Base Alpha in a rover, traveling 22.5° north of east at a speed of 15.0 kph. After 2.20 hours, a sudden rockslide destroys your rover's fuel cell, leaving you stranded. You have only 5.00 hours of oxygen remaining, and can only travel overland at a speed of 5.00 kph. Luckily, the colony has created a network of supply bunkers for just such an emergency. Checking your maps, you see that Bunker B is closest, located 50.0 km due northeast of Base Alpha.



(A) *(12 points)* How far is Bunker B from your current location? What direction should you travel, in order to proceed directly toward the bunker?

• distance travelled in rover: $d_1 = V_1 \Delta t_1 = \boxed{33.0\text{ km}}$

displacement is thus $\vec{\Delta r_1} = \langle +d_1\cos\theta_1\rangle\hat{i} + \langle +d_1\sin\theta_1\rangle\hat{j}$

$= \langle +30.49\text{ km}\rangle\hat{i} + \langle +12.63\text{ km}\rangle\hat{j}$

meanwhile, a displacement directly from Base to Bunker is $\vec{D_2} = \langle \frac{1}{\sqrt{2}}D_2\rangle\hat{i} + \langle \frac{1}{\sqrt{2}}D_2\rangle\hat{j}$

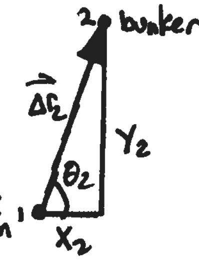
$= \langle +35.36\text{ km}\rangle\hat{i} + \langle +35.36\text{ km}\rangle\hat{j}$

Let $\vec{\Delta r_2}$ = displacement from breakdown to Bunker

require $\vec{\Delta r_1} + \vec{\Delta r_2} = \vec{D_2}$

$\Rightarrow \vec{\Delta r_2} = \vec{D_2} - \vec{\Delta r_1} = \langle +4.86\text{ km}\rangle\hat{i} + \langle +22.7\text{ km}\rangle\hat{j}$

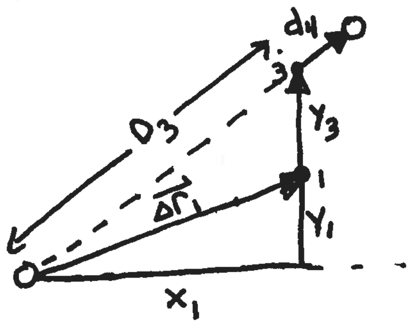
$|\vec{\Delta r_2}| = \sqrt{(x_2)^2 + (y_2)^2} = \boxed{23.2\text{ km}}$



$\tan\theta_2 = \frac{Y_2}{X_2}$

$\theta_2 = \tan^{-1}\left(\frac{22.7}{4.86}\right) = \boxed{77.9^\circ}$ North of East

(B) *(8 points)* Suppose that your gyro-compass was also destroyed in the rockslide, so you can't navigate directly toward the bunker. Your only choice is to travel north based on dead reckoning, until you are on a direct line between Base Alpha and Bunker B. At that point, you are able to pick up a directional radio signal that lies along a straight line between the two locations, and can follow the signal directly northeast to the bunker. How much time will be remaining in your oxygen supply when you reach safety? (Assume that while dead reckoning, you actually manage to travel due north.)



From position $\vec{r_1} = \langle 30.49\text{ km}, 12.63\text{ km}\rangle$

travel due north until you reach line $y = x$,

at position $\vec{r_3} = \langle 30.49\text{ km}, 30.49\text{ km}\rangle$

$\Rightarrow \vec{\Delta y_3} = (30.49\text{ km}) - (12.63\text{ km}) = \boxed{17.86\text{ km}}$

At this point, you are a distance

$D_3 = \sqrt{(30.49\text{ km})^2 + (30.49\text{ km})^2} = 43.12$ km from Base

$\Rightarrow$ distance from bunker is $d_4 = D_2 - D_3 = 6.88$ km

Total distance travelled from breakdown to bunker:

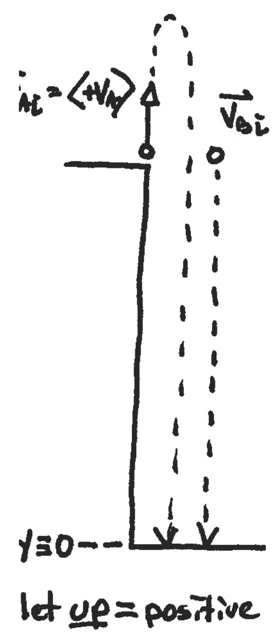
$S = |y_3| + |d_4| = 24.74$ km $\rightarrow$ at speed $V_2 = 5$ kph, time required is $\Delta t = \frac{S}{V_2} = \boxed{4.949\text{ hr}}$

$\Rightarrow 0.051$ hr $= 3.1$ min remaining O_2 supply

The following problem will be hand-graded. *__Show all your work for this problem__.* *Make no marks and leave no space on your answer card for it.*

[III] You are a settler in the first permanent human colony on Mars, where the gravitational acceleration is $g = 3.70\ \text{m/s}^2$. You are standing atop a cliff of height $H = 37.0$ m, holding an apple and a banana. You throw the apple upward off the cliff with an initial speed $v_A = 3.70$ m/s, and then after a brief delay, you drop the banana.

(12 points) What time delay between throwing the apple and dropping the banana will result in both objects striking the base of the cliff at the same time?



1st, determine time of flight experienced by each object, using

$$\vec{\Delta y} = \vec{V}_i \Delta t + \tfrac{1}{2}\langle -g\rangle \Delta t^2$$

for banana, $\vec{V}_{iB} = 0$, so we have: $\langle -H\rangle = \tfrac{1}{2}\langle -g\rangle \Delta t_B^2$

$$\Rightarrow \Delta t_B = \sqrt{\frac{2H}{g}} = \boxed{4.47\,\text{sec}}$$

for apple, it's trickier — there's a quadratic equation:

$$\langle -H\rangle = \langle +V_A\rangle \Delta t_A + \tfrac{1}{2}\langle -g\rangle \Delta t_A^2$$

$$\tfrac{1}{2} g \Delta t_A^2 - V_A \Delta t_A - H = 0$$

$$\rightarrow \Delta t_A = \frac{V_A \pm \sqrt{V_A^2 + 2gH}}{g}$$

$\Rightarrow$ choose positive root: apple strikes ground __after__ it is thrown

$$\boxed{\Delta t_A = 5.58\,\text{sec}}$$

lag: $\Delta t_B + \Delta t_{lag} = \Delta t_A \Rightarrow \Delta t_{lag} = \Delta t_A - \Delta t_B = \boxed{1.11\,\text{sec}}$

(B) *(8 points)* With what velocities will the two objects strike the base of the cliff?

easiest to use the "speed equation" $V_f^2 = V_i^2 + 2\langle -g\rangle \Delta y$

Apple: $V_{Af} = \sqrt{V_A^2 + 2(-g)(-H)} = \sqrt{V_A^2 + 2gH} = 17.0\,\text{m/s}$

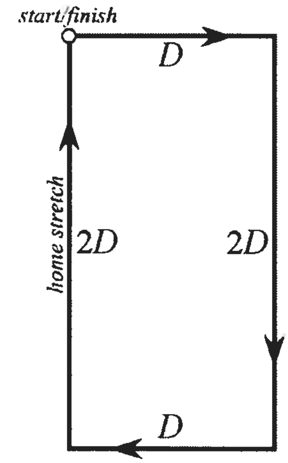
$\Rightarrow$ this is __speed__ at impact. __velocity__ is $\boxed{\vec{V}_{Af} = \langle -17.0\,\text{m/s}\rangle \text{ or } 17.0\,\text{m/s, \underline{downward}}}$

Banana: $V_{Bf} = \sqrt{0^2 + 2(-g)(-H)} = 16.5\,\text{m/s}$

as a __vector__ velocity $\boxed{\vec{V}_{Bf} = \langle -16.5\,\text{m/s}\rangle = 16.5\,\text{m/s, \underline{downward}}}$

The following problem will be hand-graded. <u>Show all your work for this problem</u>. Make no marks and leave no space on your answer card for it.

[III] You are a settler in the first permanent human colony on Mars. In the gymnasium module one day, you decide to have a footrace with another colonist. The track consists of a rectangular loop of dimension D x $2D$. you maintain a steady speed v_o along the first three legs of the loop, then speed up to a speed $2v_o$ on the final stretch. Your opponent maintains a speed $3v_o/2$ for the first three legs, then becomes winded and can only maintain a speed $v_o/2$ for the last leg.



(12 points) Who wins the race? By how much time does the winner beat the loser? (Express your answer entirely in terms of D and v_o.)

On any individual leg, motion is uniform:

$\vec{\Delta r} = \vec{v}\Delta t$ or $|\vec{\Delta r}| = |\vec{v}|\Delta t \longrightarrow d_i = v_i \Delta t_i$

$\Rightarrow \Delta t_i = \frac{d_i}{v_i}$ and $\Delta t_{TOT} = \sum_i \Delta t_i$

You: $\Delta t = \frac{D}{V_o} + \frac{2D}{V_o} + \frac{D}{V_o} + \frac{2D}{2V_o} = \frac{D}{V_o}[1+2+1+1] = \boxed{\frac{5D}{V_o} = \Delta t_A}$

Opponent: $\Delta t = \frac{D}{3/2 V_o} + \frac{2D}{3/2 V_o} + \frac{D}{3/2 V_o} + \frac{2D}{1/2 V_o} = \frac{D}{V_o}\left[\frac{2}{3} + \frac{4}{3} + \frac{2}{3} + 4\right] = \boxed{\frac{20D}{3V_o} = \Delta t_B}$

To compare, find common denominator: $\Delta t_A = \frac{15D}{3V_o} < \frac{20D}{3V_o} = \Delta t_B$

Your time is faster, so you win! In fact, you win by $|\Delta t_A - \Delta t_B| = \boxed{\frac{5D}{3V_o}}$

(8 points) What was your average speed and average velocity for the entire race? What was your opponent's average speed and velocity?

Both runners started and ended at the same location

$\Rightarrow$ net displacements are $\vec{\Delta r}_A = \vec{\Delta r}_B \equiv 0$

For both runners, $\boxed{\vec{V}_{av} = \frac{\vec{\Delta r}}{\Delta t} \equiv 0}$

average speed though is $V_{av} = \frac{\text{total distance}}{\text{elapsed time}} = \frac{6D}{\Delta t_{TOT}}$

$\Rightarrow V_A = 6D / \frac{5D}{V_o} = \frac{6}{5} V_o \Rightarrow \boxed{V_{A,av} = \frac{6}{5}V_o}$

$V_B = 6D / \frac{20D}{3V_o} = \frac{18}{20} V_o \quad \boxed{V_{B,av} = \frac{9}{10}V_o}$

Question value 8 points

(1) In the following expression, the symbol d represents distance, t represents time, v represents speed, and a represents acceleration magnitude. Which of the formulas below is dimensionally correct?

(a) $v^2/d = 3at$

(b) $v_f - v_i = \frac{1}{2}at^2$

(c) $v_B^2 - v_A^2 = 2a^2t$

(d) $(a_A^2 - v_A^2t^2)/d_A = v_B^2/d_B^2$

$\boxed{\text{(e)} \quad 5va = d^2/t^3}$

(a) $v^2/d \to \frac{[L/T]^2}{[L]} = \left[\frac{L^2}{T}\right]$ $\quad 3at \to \left[\frac{L}{T^2}\right][T] = [L/T]$

$\to$ $\boxed{\text{NOT A MATCH}}$

(b) $V_f - V_i \to [L/T]$ $\quad \frac{1}{2}at^2 \to \left[\frac{L}{T^2}\right][T]^2 = [L]$

$\to$ $\boxed{\text{NOT A MATCH}}$

(c) $V_B^2 - V_A^2 \to [L/T]^2$ $\quad 2a^2t \to [L/T^2]^2[T] = [L^2/T^3]$

$\to$ $\boxed{\text{NOT A MATCH}}$

(d) $(a_A^2 - V_A^2t^2)/d_A \to \left[[L/T^2]^2 - \left[\frac{L}{T}\right]^2[T]^2\right]/[L] = \left[\frac{[L^2/T^4] - [L^2]}{[L]}\right]$

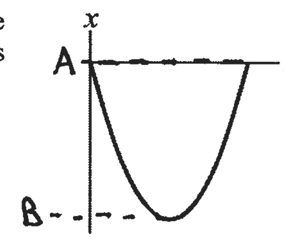
left-hand side does not even match itself!

(e) $5va \to [L/T][L/T^2] = \left[\frac{L^2}{T^3}\right]$ $\quad d^2/t^3 \to [L]^2/[T]^3 = \left[\frac{L^2}{T^3}\right]$

$\boxed{\text{DIMENSIONS ARE A MATCH}}$

Question value 8 points

(2) A cart is given an initial shove up an inclined ramp. The cart starts at A, coasts up the track and stops at B, and then returns back down the track to A. An observer collects position data and constructs the x-vs-t graph shown at right.



At what point (if any) during the motion does the acceleration of the cart have a *negative* value?

(a) The acceleration is negative *only* as it moves from A to B.

(b) The acceleration is negative while it is moving from A to B, and from B to A, but *not* at the moment that it is stopped at B.

$\boxed{\text{(c) At no point during the cart's motion is the acceleration negative.}}$

(d) At all points during the cart's motion the acceleration is negative.

(e) The acceleration is negative *only* as it moves from B to A.

From the x-vs-t graph, it is clear that "up the ramp" = negative

So: at what times is $\vec{a}$ directed up the ramp?

Answer: $\boxed{\text{NEVER}}$ $\vec{a}$ is always directed $\boxed{\text{down the ramp}}$ ($\boxed{\text{positive}}$)

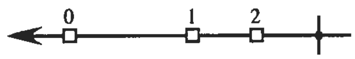
Also note: Calculus is our friend here: $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$

- accel is 2nd derivative of position
- in calc, 2nd derivative is found as "curvature" or "concavity"
- graph is clearly concave up — that implies $\boxed{\text{accel is positive}}$

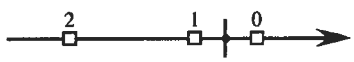
Question value 8 points

(3) Each of the motion diagrams below has a coordinate system with a directional arrow denoting the "positive" direction and a crosshair indicating the origin. In which of the diagrams does position "1" correspond to the object having a $\boxed{\text{positive velocity}}$ and a $\boxed{\text{negative acceleration?}}$

(a)  $\vec{V}$ is positive (✓), speed is increasing (X)

(b)  $\vec{V}$ is negative (X), speed is decreasing (✓)

$\boxed{\text{(c)}}$  $\vec{V}$ is positive (✓), speed is decreasing (✓)

(d)  $\vec{V}$ is negative (X), speed is increasing (X)

(e)  $\vec{V}$ is negative (X), speed is increasing (X)

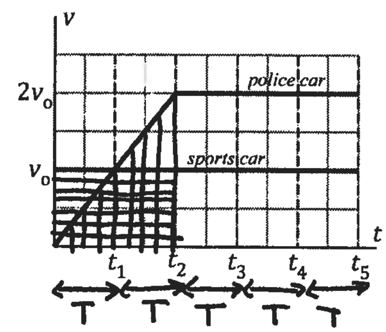
Note: "positive velocity and negative acceleration" means: $\boxed{\vec{a} \text{ is opposite to } \vec{V}}$

$\Downarrow$

$\boxed{\underline{\text{speed}} \text{ is decreasing}}$

Question value 8 points

(4) A stationary police car is passed by a speeding sportscar travelling with speed v_o. After a brief delay, the police car gives chase, accelerating uniformly to speed $2v_o$ and then maintaining that speed. Both cars' velocities are graphed at right. At what time will the police car overtake the sportscar?



(a) At time t_3.

$\boxed{\text{(b) At time } t_2.}$

(c) At time t_5.

(d) At time t_1.

(e) At time t_4.

"Police car catches up"

$\Downarrow$

Same <u>total</u> displacement for <u>both</u> cars

let $T = (t_2 - t_1) = (t_3 - t_2) = (t_4 - t_3) = (t_5 - t_4)$

(= "two squares" along time axis)

• at t_1 : $\overrightarrow{\Delta X}_{sc} = V_o T$ (rectangle) $\overrightarrow{\Delta X}_{pc} = \frac{1}{2}(V_o)(T)$ (triangle)

$\overrightarrow{\Delta X}_{pc} = \frac{1}{2}\overrightarrow{\Delta X}_{sc}$ → not yet!

• at t_2 : $\overrightarrow{\Delta X}_{sc} = V_o(2T) = 2V_oT$ $\overrightarrow{\Delta X}_{pc} = \frac{1}{2}(2V_o)(2T) = 2V_oT$

$\boxed{\overrightarrow{\Delta X}_{pc} = \overrightarrow{\Delta X}_{sc}}$ police car overtakes sports car $\boxed{\text{at time } t_2}$

Question value 8 points

(5) In coordinate system S, a turtle has a position vector given by $\vec{r}_S = \langle 2\text{ m}, 6\text{ m}\rangle$. Coordinate system T has axes parallel to coordinate system S, but its origin is located at position $\vec{O}_S = \langle 3\text{ m}, 2\text{ m}\rangle$ in coordinate system S. What is the position vector of the turtle in coordinate system T?

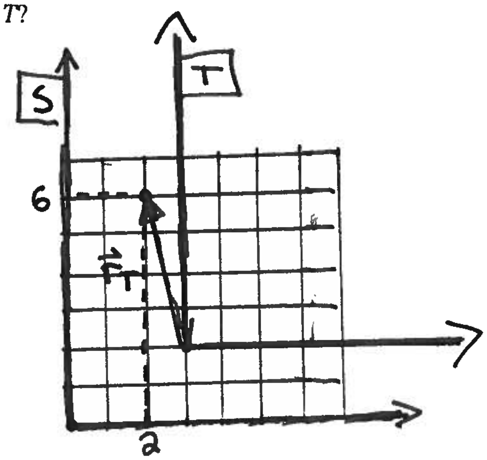
(a) $\vec{r}_T = \langle 5\text{ m}, 8\text{ m}\rangle$

(b) $\vec{r}_T = \langle 1\text{ m}, 4\text{ m}\rangle$

(c) $\boxed{\vec{r}_T = \langle 1\text{ m}, -4\text{ m}\rangle}$

(d) $\vec{r}_T = \langle -1\text{ m}, -4\text{ m}\rangle$

(e) $\vec{r}_T = \langle -1\text{ m}, 4\text{ m}\rangle$



from figure, it should be clear that $\langle +2, +6\rangle_S \longleftrightarrow \langle -1, +4\rangle_T$