Physics 2211 BCD
Spring 2015
Test 2

Test form 422

Name Official Solution Set

Recitation Section (see back of test):________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

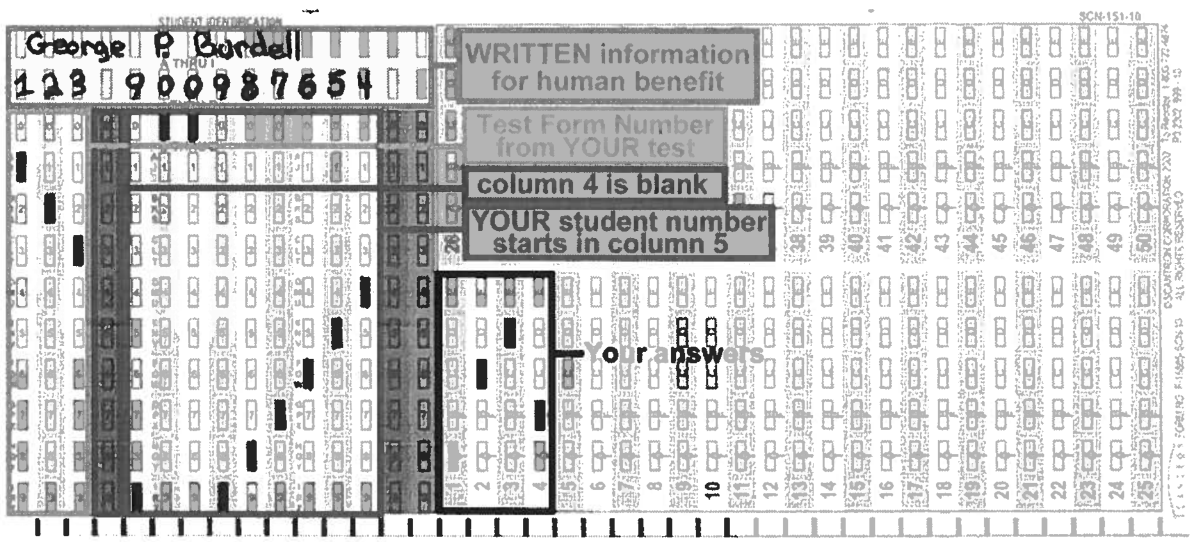
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

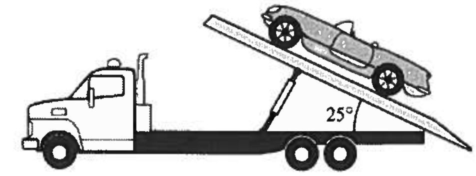
Your test form is: 422



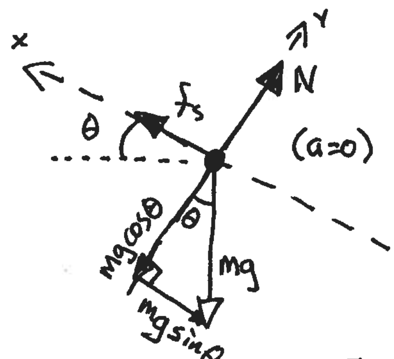
Our next test will be on Monday, March 9!

The following problem will be hand-graded. *<u>Show all your work for this problem</u>.* *Make no marks and leave no space on your answer card for it.*

[I] A flatbed wrecker is hauling a car, travelling west at a constant speed $v_0 = 16$ m/s. Due to an engineering flaw, the hydraulics engage while the truck is in motion, tilting the flatbed up to an angle $\theta = 25°$ above the horizontal. To make matters worse, the straps anchoring the car to the flatbed have sheared off, leaving only friction to hold the car on the wrecker! The friction coefficients between the wheels and the bed are $\mu_s = 0.67$ and $\mu_k = 0.40$.



(A) *(10 points)* Draw a free body diagram <u>for the car</u>, which does not slip as the wrecker continues moving at speed v_0. Use Newton's laws to determine the friction force acting on the car. Express your answer as a multiple of mg, the gravitational force on the car. <u>You will be graded on the quality of your diagram as much as the validity of your final answer!</u>



① $\vec{v}$ = constant, so $\vec{a}=0$: car is in equilibrium

② Tendency for car to slip down ramp: $\boxed{\text{friction points up ramp}}$

let "up the ramp" be positive x-direction

$$\Sigma \vec{F}_x = m\vec{a}_x$$

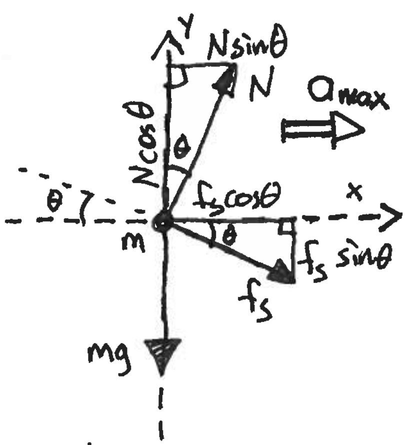
$$\langle +f_s\rangle + \langle -mg\sin\theta\rangle = 0 \Rightarrow \boxed{f_s = mg\sin\theta = 0.42\, mg}$$

[Note that "$\mu_s N$" is not appropriate here, because there is no guarantee that static friction is at its upper limit]

(B) *(10 points)* The driver of the wrecker realizes his predicament, and quickly brakes to a stop. Determine the maximum braking acceleration that the wrecker have without allowing the car to slip forward and launch itself off the inclined truck bed? Draw a new free-body diagram <u>for the car</u> as it accelerates to a stop along with the truck. Be sure to draw the acceleration vector for the car, because it will impact how you use Newton's laws to find a_{max}.

① Slowing down: acceleration is horizontal, to the right — decompose using horizontal and vertical axes

② At max accel, car is about to slip up ramp — $f_s = \boxed{\text{max}}$, points down slope $= \mu_s N$



• $$\Sigma \vec{F}_y = 0 = \langle +N\cos\theta\rangle + \langle -f_s\sin\theta\rangle + \langle -mg\rangle$$

$$N\cos\theta - (\mu_s N)\sin\theta = mg \rightarrow N = \frac{mg}{\cos\theta - \mu_s\sin\theta}$$

• $$\Sigma \vec{F}_x = m\vec{a}_x$$

$$\langle +N\sin\theta\rangle + \langle +f_s\cos\theta\rangle = m\langle +a_{max}\rangle$$

$$N\sin\theta + (\mu_s N)\cos\theta = m\,a_{max}$$

$$N(\sin\theta + \mu_s\cos\theta) = m\,a_{max}$$

$$\left(\frac{mg}{\cos\theta - \mu_s\sin\theta}\right)(\sin\theta + \mu_s\cos\theta) = m\,a_{max}$$

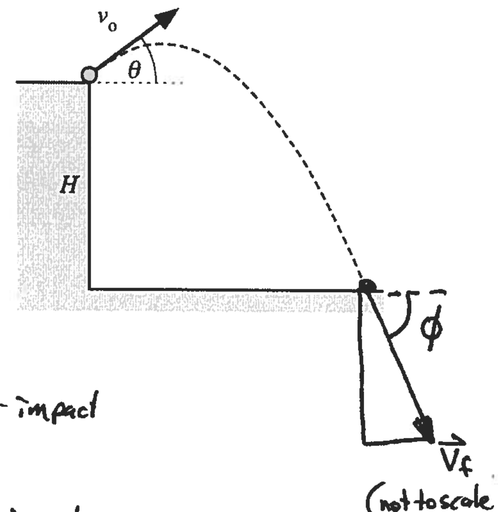
$$\boxed{a_{max} = g\left[\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right] = 1.65g = 16\ m/s^2}$$

Vert/Horiz axes chosen so that a_{max} appears only in one equation: $\Sigma\vec{F}_x = m\vec{a}_x$

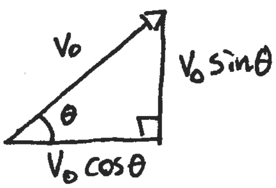
The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

[II] You are standing atop a cliff of unknown height H. You throw a stone off the cliff with a speed $v_o = 15.0$ m/s, directed at an angle of 36.9° above the horizontal. You time the flight of the stone, finding that it takes a total time $\Delta t_v = 3.25$ seconds to strike the valley floor below.

(15 points) What is the velocity of the stone at the moment it hits the valley floor?



Decompose initial velocity:



for $\theta = 36.9°$ [3-4-5 triangle]:

$\sin\theta = \frac{3}{5}$ $\cos\theta = \frac{4}{5}$ $\tan\theta = \frac{3}{4}$

so: $\boxed{V_{0x} = \frac{4}{5}V_0 = \text{constant}} \rightarrow V_{xf} = \frac{4}{5}V_0$ at impact

also, $V_{0y} = \frac{3}{5}V_0$

$\rightarrow$ vertical free-fall tells us: $\vec{V}_{yf} = \vec{V}_{y0} + \langle -g \rangle \Delta t$

so, at impact, $\langle - V_{yf} \rangle = \langle +\frac{3}{5}V_0 \rangle + \langle -g \rangle \Delta t_v$ at valley floor

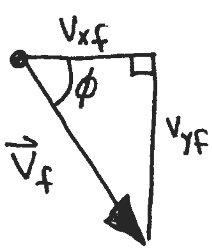
down ← magnitude: "V_{yf}" denotes a positive value

$\Rightarrow V_{yf} = g\Delta t_v - \frac{3}{5}V_0 = 31.9 \text{ m/s} - 9.0 \text{ m/s} = \boxed{22.9 \text{ m/s}}$

$V_{xf} = V_{x0} = \frac{4}{5}V_0 = \boxed{12.0 \text{ m/s}}$ in Cartesian form:

$\boxed{\vec{V}_f = \langle +12.0 \text{ m/s} \rangle \hat{i} + \langle -22.9 \text{ m/s} \rangle \hat{j}}$

Cartesian form is an acceptable final answer



$|\vec{V}_f| = \sqrt{V_{xf}^2 + V_{yf}^2} = \boxed{25.8 \text{ m/s}}$

$\phi = \tan^{-1}\left(\frac{V_{yf}}{V_{xf}}\right) = \boxed{62.3° \text{ below the horizontal}}$ magnitude and direction is also a valid final answer

Question value 5 points — mark your answer in space #1 on the answer card.

(01) What is the height H of the cliff?

(a) $H = 12.8$ m

(b) $H = 29.3$ m

(c) $H = 22.5$ m

(d) $H = 51.8$ m

(e) $H = 3.01$ m

equation for vertical motion:

$\vec{\Delta y} = \vec{V}_{0y}\Delta t + \frac{1}{2}\langle -g \rangle \Delta t^2$

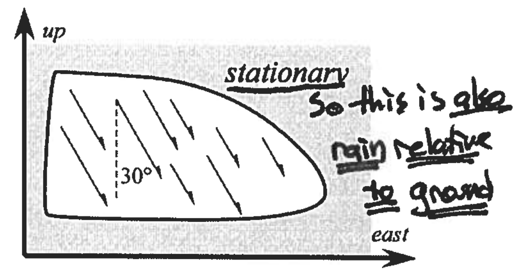
$\langle -H \rangle = \langle +V_0\sin\theta \rangle \Delta t_v + \frac{1}{2}\langle -g \rangle \Delta t_v^2$

$\boxed{H = -V_0\sin\theta\,\Delta t + \frac{1}{2}g\Delta t_v^2 = 22.5\text{m}}$

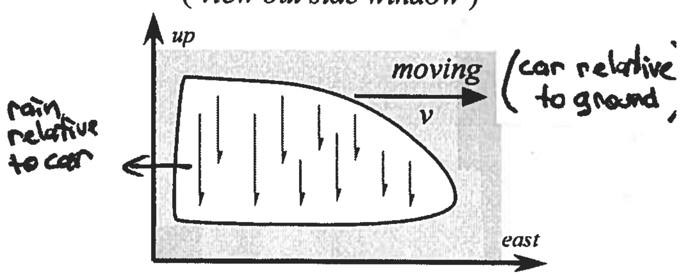
The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[III] You are driving your car in a rainstorm. When stopped at a traffic light and facing eastward, you see the rain fall *forward* (i.e. also eastward), making an angle $\theta = 30.0°$ relative to the vertical (top figure). When the light turns green, you begin travelling east, and find that when your speed is $v = 36$ kph, the rain appears to be falling straight down (bottom figure).

(15 points) If you turn your car around, and travel *westward* at the same speed v, what angle will the raindrops appear to fall, relative to the vertical?



(view out side window)



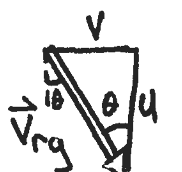
When driving East:

car relative to ground: $\vec{V}_{cg} = \langle +v \rangle \hat{i}$

rain relative to car $\vec{V}_{rc} = \langle -u \rangle \hat{j}$

[speed v is known, speed u is unknown]

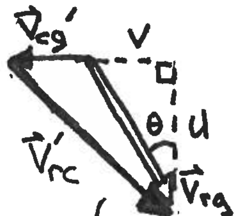
→ rain relative to ground:



$\vec{V}_{rg} = \vec{V}_{rc} + \vec{V}_{cg} = \langle -u \rangle \hat{j} + \langle +v \rangle \hat{i}$

$\Rightarrow \boxed{\vec{V}_{rg} = \langle +v \rangle \hat{i} + \langle -u \rangle \hat{j}}$ where u and v are related by $\boxed{\tan\theta = \frac{v}{u}}$

Now — travel west at speed v: $\vec{V}_{cg}' = \langle -v \rangle \hat{i}$

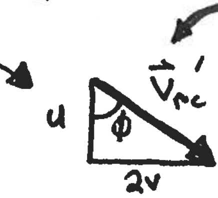


$\vec{V}_{rg}$ does not change, still $= \langle +v \rangle \hat{i} + \langle -u \rangle \hat{j}$

New $\vec{V}_{rc}'$ is found from $\vec{V}_{rg} = \vec{V}_{rc}' + \vec{V}_{cg}'$

$\vec{V}_{rc}' = \vec{V}_{rg} - \vec{V}_{cg}' = [\langle +v \rangle \hat{i} + \langle -u \rangle \hat{j}] - [\langle -v \rangle \hat{i}]$

$\boxed{\vec{V}_{rc}' = \langle 2v \rangle \hat{i} + \langle -u \rangle \hat{j}}$



direction relative to vertical

$\phi = \tan^{-1}\left(\frac{2v}{u}\right) = \tan^{-1}\left(2\frac{v}{u}\right) = \tan^{-1}(2\tan\theta) = \boxed{49.1°}$

Note: values of u, v are irrelevant!

Question value 5 points — mark your answer in space #2 on the answer card.

(02) Although the rain is actually falling east of vertical, if you travel east quickly enough, the rain will appear to fall west of vertical (i.e "backwards", from your perspective in the car). At what eastward speed will the rain appear to fall 30° west of vertical? [Hint a good vector diagram makes a big difference here!]

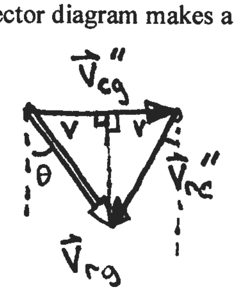
(a) 24 kph

(b) 18 kph

(c) 48 kph

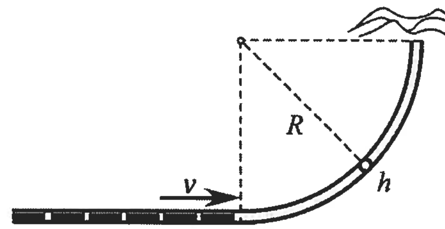
$\boxed{\text{(d) 72 kph}}$

(e) 62 kph



Note that $\vec{V}_{rg}$, $\vec{V}_{rc}''$, and $\vec{V}_{cg}''$ all form an equilateral triangle, having length 2v

A train traveling east through the mountains enters a northward bend with radius of curvature R. Just as it enters the curve with speed v, the satellite uplink indicates that an avalanche has blocked the track, 90° around the bend. The train must brake to a stop before it rounds the bend, or it will be derailed by the debris field.



Question value 5 points

(03) What is the magnitude of the tangential acceleration that will stop the train just as it reaches the debris?

~~(a) $a_{tan} \geq v^2 \cdot 2\pi$~~

$\boxed{\text{(b)} \quad a_{tan} \geq v^2/\pi R}$

~~(c) $a_{tan} \geq v/R$~~

(d) $a_{tan} \geq v^2/2R$

~~(e) $a_{tan} \geq 2v/\pi R$~~

These three answers all have WRONG PHYSICAL DIMENSION to be "acceleration"

• Tangential stopping distance: $\frac{1}{4}$ circle, $\Delta s = \frac{2\pi R}{4} = \frac{\pi}{2}R$

• Tangential speed equation:

$$\cancel{V_f^2} = V_i^2 + 2\langle -a_{tan}\rangle\langle \Delta s\rangle$$

0

acceleration is opposite to direction of travel!

$$0 = V^2 - \cancel{2}a_{tan}\cdot\frac{\pi}{\cancel{2}}R$$

$$\boxed{a_{tan} = \frac{V^2}{\pi R}}$$

(04) Consider the train when it is halfway through the turn, at poisiton h in the diagram above. What will be the train's *angular* speed ω around the bend, at that point?

Ack! I can't splell!

$\boxed{\text{(a)} \quad \omega_h = v\,/\,\sqrt{2}R}$

~~(b) $\omega_h = 2v\,/\,\pi$~~

(c) $\omega_h = v\,/\,2R$

~~(d) $\omega_h = \pi R\,/\,4v$~~

(e) $\omega_h = 4v\,/\,\pi R$

again: WRONG PHYSICAL DIMENSION

use accel above, and a second application of speed equation, now with $\Delta s = \frac{\pi}{4}R$:

$$V_h^2 = V^2 + 2\left\langle -\frac{V^2}{\pi R}\right\rangle\left\langle +\frac{\pi}{4}R\right\rangle$$

$$= V^2 - \frac{1}{2}V^2$$

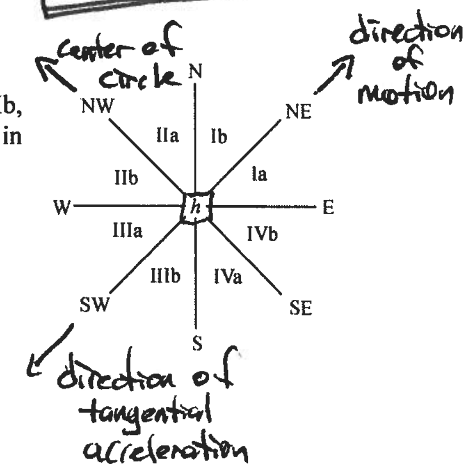
$$V_h^2 = \frac{1}{2}V^2 \rightarrow V_h = \frac{V}{\sqrt{2}}$$

angular speed is $\omega_h = \frac{V_h}{R}$

so $\boxed{\omega_h = \frac{V}{\sqrt{2}R}}$

Question value 5 points

(05) In the figure at right, the eight cardinal compass directions split the map into octants: Ia/Ib, IIa/IIb, IIIa/IIIb, and IVa/IVb. When the train is at position h, halfway through the turn, in which octant will its total acceleration vector lie?



~~(a) $\vec{a}_h$ lies in octant IVa.~~

(b) $\vec{a}_h$ lies in octant IIb.

~~(c) $\vec{a}_h$ lies in octant IIIb.~~

(d) $\vec{a}_h$ lies in octant IIIa.

~~(e) $\vec{a}_h$ lies in octant Ib.~~

At position h

• tangential accel is SW

$$a_{tan} = \frac{V^2}{\pi R}$$

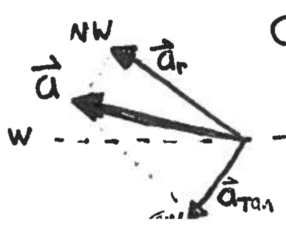
• radial accel is NW

$$a_r = a_{cent} = \frac{V_h^2}{R} = \frac{V^2}{2R}$$

Total accel must lie between these components → Note that $a_r = \frac{V^2}{2R} = \frac{\pi}{2}\cdot\frac{V^2}{\pi R} = \frac{\pi}{2}a_{tan}$

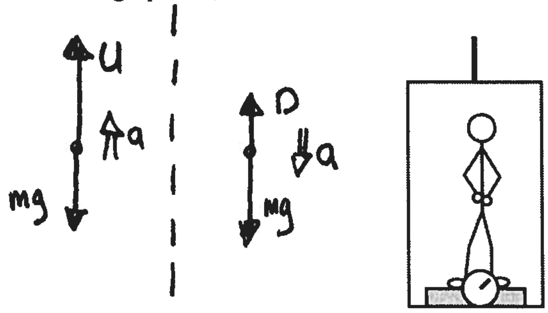
$a_r \approx 1.57\,a_{tan} \Rightarrow a_r > a_{tan}$

→ $\vec{a}$ is between NW and SW, somewhere only octants IIb or IIIa are possible



$\boxed{\vec{a} \text{ lies in octant IIb}}$

Bob stands on a spring scale in an express elevator Starting from ground level at $t = 0$, the elevator accelerates smoothly upward (with constant magnitude a) until it is halfway to the penthouse, and which point it accelerates smoothly to a stop (with the same constant magnitude a) right at the penthouse. While accelerating upward, the scale reads a force $U = 960$ N. While acclerating downward, the scale reads a force $D = 720$ N.



Question value 5 points

(06) What is Bob's *perceived weight* during the first stage of the ascent?

$\boxed{\text{(a) 960 N}}$

(b) 720 N

(c) "Perceived weight" is undefined for an *accelerating* body.

(d) 840 N

(e) 240 N

Perceived weight is, by definition, the force pushing in opposition to gravity, the actual scale reading, $\boxed{960N}$

Question value 5 points

(07) What is Bob's *true weight* (i.e., the graviational force *mg* acting on Bob) during the first stage of the ascent?

(a) 960 N

(b) 840 N

(c) 240 N

(d) 480 N

(e) 720 N

while accelerating up: $\langle +U \rangle + \langle -mg \rangle = m \langle +a \rangle$

while accelerating down: $\langle +D \rangle + \langle -mg \rangle = m \langle -a \rangle$

⇒ add equations: $D + U - 2mg = +ma - ma = 0$

$D + U = 2mg$

$$\boxed{mg = \frac{D+U}{2} = 840N}$$

Question value 5 points

(08) What is the magnitude of the elevator's acceleration, during either stage of the ascent?

(a) $a = 2.5\ \text{m/s}^2$

(b) $a = 9.8\ \text{m/s}^2$

(c) $a = 14\ \text{m/s}^2$

(d) $a = 4.9\ \text{m/s}^2$

(e) $a = 1.4\ \text{m/s}^2$

From above,

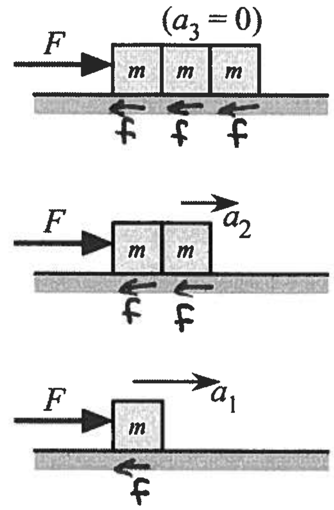
$$\begin{array}{r} U - mg = ma \\ -\left[D - mg = -ma\right] \\ \hline U - D \cancel{-mg+mg} = 2ma \end{array}$$

$$a = \frac{U-D}{2m} = g\,\frac{U-D}{2mg} = g\,\frac{U-D}{2\left[\frac{U+D}{2}\right]}$$

$$a = g\,\frac{U-D}{U+D} = \frac{240N}{1680N} \cdot g$$

$$\boxed{a = \frac{g}{7} = 1.4\ \text{m/s}^2}$$

Three identical blocks are placed on a *rough* horizontal surface. When a horizontal force F is applied to the blocks (top), they move along the surface at constant speed. When one block is removed (middle), the force F is sufficient to impart an acceleration of magnitude a_2 to the two remaining blocks. When a second block is removed (bottom), the force F imparts an acceleration of magnitude a_1 to the last remaining block.



let friction force between one block and surface be "f"

Question value 5 points

(09) Qualitatively, compare the acceleration a_1 to the acceleration a_2.

(a) $a_1 = a_2$

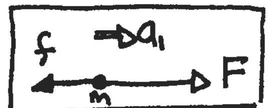
$\boxed{\text{(b)} \quad a_1 > 2a_2}$

(c) $a_2 < a_1 < 2a_2$

(d) $a_1 = 2a_2$

(e) $0 < a_1 < a_2$

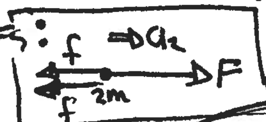
One block:



$\langle +F \rangle + \langle -f \rangle = m \langle +a_1 \rangle$

note that net force is $\Sigma \vec{F}_1 = \langle +F - f \rangle$

Two blocks:



note that net force is $\Sigma \vec{F}_2 = \langle +F - 2f \rangle$

$\langle +F \rangle + \langle -f \rangle + \langle -f \rangle = (2m) \langle +a_2 \rangle$

Observe that $\Sigma \vec{F}_1 > \Sigma \vec{F}_2$

To compare qualitatively

for one block: half the mass $\boxed{\text{AND}}$ greater net force : $\boxed{a_1 > 2a_2}$

did you miss this bit?

Question value 5 points

(10) If $a_1 = 0.3\ g$, what is the coeffecient of friction between the blocks and the surface?

(a) $\mu_k = 0.10$

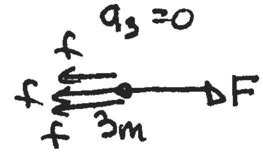
(b) $\mu_k = 0.20$

(c) $\mu_k = 0.05$

$\boxed{\text{(d)} \quad \mu_k = 0.15}$

(e) $\mu_k = 0.30$

Three blocks:



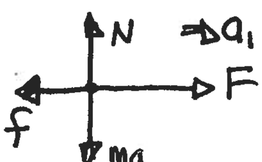
$\Sigma \vec{F}_x = m\vec{a}_x$

$\langle +F \rangle + \langle -f \rangle + \langle -f \rangle + \langle -f \rangle = 0$

$F = 3f$

or $f = \frac{F}{3}$

go back to one block:



$\Sigma \vec{F}_x = m\vec{a}_x$

$\langle +F \rangle + \langle -f \rangle = m \langle +a_1 \rangle$

$3f - f = m \cdot \frac{3}{10} g$

$2f = \frac{3mg}{10} \Rightarrow f = \frac{3mg}{20}$

$\Sigma \vec{F}_y = m\vec{a}_y$

$\langle +N \rangle + \langle -mg \rangle = 0$

$N = mg$

if $f = f_k = \mu_k N = \mu_k (mg)$, we see:

$\mu_k mg = \frac{3}{20} mg$ $\boxed{\mu_k = \frac{3}{20} = 0.15}$