Physics 2211 BCD
Spring 2015
Test 4

Test form **444**

Name Official Solution Set

Recitation Section (see back of test): ________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

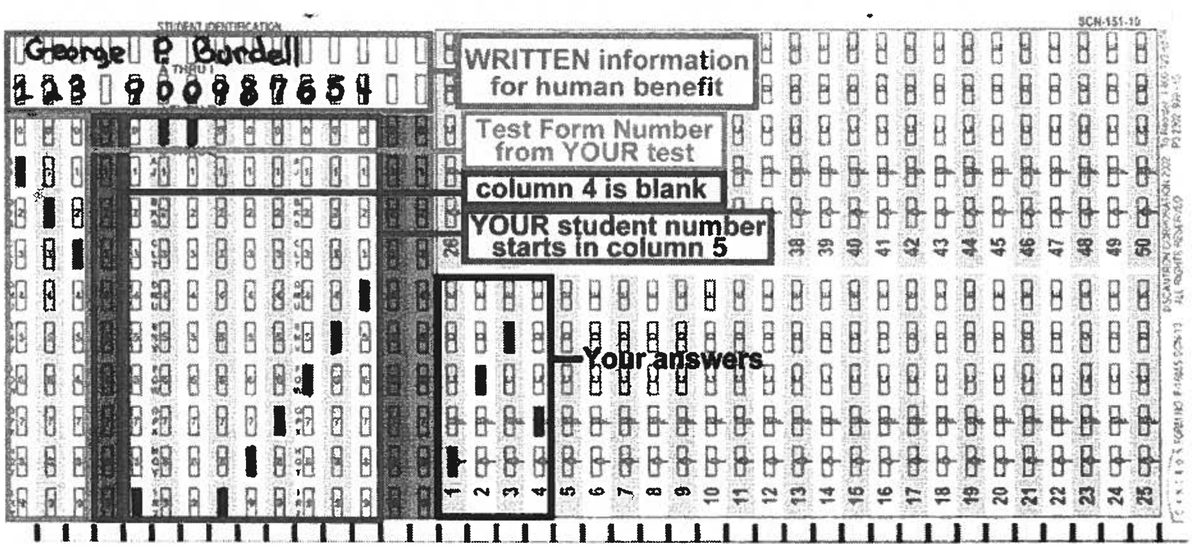
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

Your test form is: 444



Our Final Exam will be on Monday, April 27 at 8:00am!

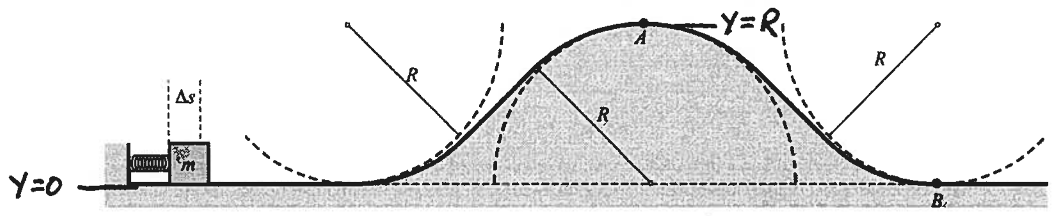
The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

[I] A block of mass m lies on a frictionless horizontal surface. It is placed against a spring having elastic constant k, which is then compressed by an unspecified distance Δs and then released from rest. After it leaves the spring, the block passes over a semicircular hill having radii of curvature R (with upward curvature at the bottom of the hill, and downward curvature at the top).

(A) *(10 points)* What maximum compression distance Δs_{max} will allow the block to travel over the top of the hill, without becoming airborne at the (downcurving) apex of the hill, point A? (Hint: what would be the *forces* acting on the block, at the moment it becomes airborne?)

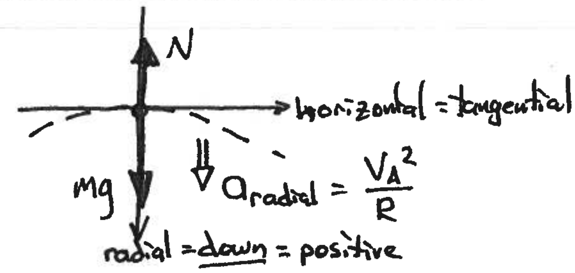
from i to A:
energy is conserved
$E_i = E_A$

$$\boxed{\tfrac{1}{2}k\Delta s^2 = mgR + \tfrac{1}{2}mV_A^2}$$



goal is to figure out speed at A
→ invoke 2nd Law: block is following a circular trajectory

$$\sum \vec{F}_r = m\vec{a}_r \Rightarrow \langle -N\rangle + \langle +mg\rangle = m\left\langle +\frac{V_A^2}{R}\right\rangle$$



We are considering limit where block loses contact:

$\boxed{N \to 0}$ so that $mg = m\frac{V_A^2}{R} \Rightarrow mV_A^2 = mgR$

$\boxed{\tfrac{1}{2}mV_A^2 = \tfrac{1}{2}mgR}$ put this result back into the energy expression

$$\Rightarrow \tfrac{1}{2}k\Delta s^2 = mgR + \tfrac{1}{2}mgR = \tfrac{3}{2}mgR$$

$$\Delta s^2 = \frac{3mgR}{k} \Rightarrow \boxed{\Delta s = \sqrt{\frac{3mgR}{k}}}$$

for any greater Δs, block will be moving so fast at A that it will become airborne

(B) *(10 points)* Suppose instead that the spring is compressed so that the block will only *just barely* make it over the top of the hill without stopping. What will be the apparent weight of the block as it reaches the (upcurving) base of the hill, just before it reaches point B—while still ***on*** the hill. (Recall that apparent weight is found from the normal force acting on the block, at the point in question.)

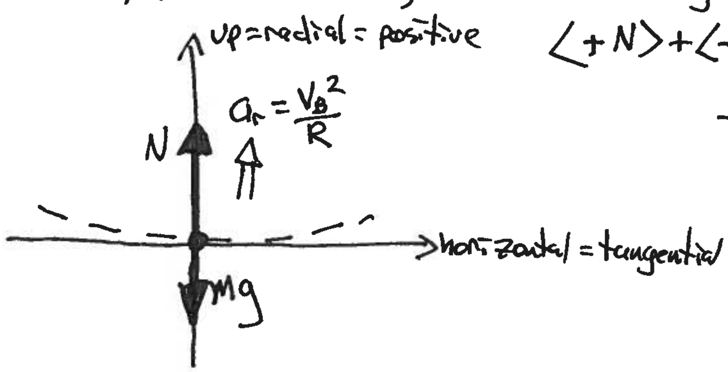
"block barely makes it over hill" → implies that now, $K_A = \tfrac{1}{2}mV_A^2 \simeq 0$

So - conservation of energy from A to B: $\cancel{K_A}^{\,0} + U_A = K_B + \cancel{U_B}^{\,0}$

$$mgR = \tfrac{1}{2}mV_B^2 \rightarrow \boxed{V_B^2 = 2gR}$$

Now, just before B, block is following an upcurving circular path

$$\langle +N\rangle + \langle -mg\rangle = m\left\langle +\frac{V_B^2}{R}\right\rangle = m\langle +2g\rangle$$

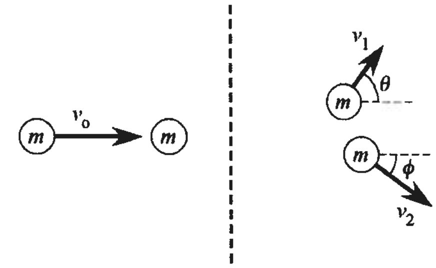


$$\Rightarrow \boxed{N = 3mg}$$

Apparent weight is three times the block's true weight

The following problem will be hand-graded. <u>*Show all your work for this problem.*</u> *Make no marks and leave no space on your answer card for it.*

[II] A puck of mass m is sliding on frictionless ice with speed v_o when it collides with an idential puck having the same mass m. Assume that the collision is <u>perfectly elastic</u>. After the collision, the first puck is observed to travel away from the collision at some angle θ, moving with a speed $v_1 = 0.600\ v_o$, while the second puck departs at some angle ϕ, moving with some speed v_2.



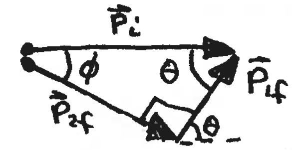
(15 points) Determine the speed v_2 of the second puck after the collision (expressed as a *decimal* fraction or multiple of v_o), and then determine the two deflection angles θ and ϕ for the pucks, expressed as <u>numerical values</u> relative to the first puck's initial direction of travel.

① "Perfectly Elastic" → no KE loss: $K_i = K_f \rightarrow \frac{1}{2}mv_o^2 = \frac{1}{2}m(0.6v_o)^2 + \frac{1}{2}mv_2^2$

$v_2^2 = v_o^2 - 0.36v_o^2 = 0.64v_o^2 \Rightarrow \boxed{v_2 = 0.800v_o}$

② 2D momentum conservation $\vec{P}_i = \vec{P}_{1f} + \vec{P}_{2f}$ → $\boxed{\text{Quick way:}}$ $|\vec{P}_i| = \frac{5}{5}mv_o$, and $|\vec{P}_{1f}| = \frac{3}{5}mv_o$, $|\vec{P}_{2f}| = \frac{4}{5}mv_o$

$\boxed{\text{This is a 3-4-5 right triangle!}}$



$\cos\phi = \frac{|\vec{P}_{2f}|}{|\vec{P}_i|} = \frac{4}{5}$ $\boxed{\phi = 36.9^\circ}$

$\cos\theta = \frac{|\vec{P}_{1f}|}{|\vec{P}_i|} = \frac{3}{5}$ $\boxed{\theta = 53.1^\circ}$

$\boxed{\text{"Long" way: by components}}$

x: $\langle +mv_o \rangle = \langle +mv_1\cos\theta \rangle + \langle +mv_2\cos\phi \rangle$

$1 = \frac{3}{5}\cos\theta + \frac{4}{5}\cos\phi$

Ⓐ $\boxed{5 - 3\cos\theta = 4\cos\phi}$

y: $0 = \langle +mv_1\sin\theta \rangle + \langle -mv_2\sin\phi \rangle \rightarrow 0 = \frac{3}{5}\sin\theta - \frac{4}{5}\sin\phi$

Ⓑ $\boxed{3\sin\theta = 4\sin\phi}$

we now have two equations in ϕ, ϕ

• take Ⓐ^{2} + Ⓑ^{2}:

$25 - 30\cos\theta + 9\cos^2\theta = 16\cos^2\phi$

$+ \quad 9\sin^2\theta = 16\sin^2\phi$

$25 - 30\cos\theta + 9 = 16$

$\cos\theta = \frac{3}{5}$ $\boxed{\theta = 53.1^\circ}$

then $\sin\theta = \frac{4}{5}$

so Ⓑ becomes: $3 \cdot \frac{4}{5} = 4\sin\phi \rightarrow \sin\phi = \frac{3}{5}$ $\boxed{\phi = 36.9^\circ}$

Question value 5 points — mark your answer in space #1 on the answer card.

(1) Suppose instead that the collision between the two pucks is perfectly <u>*inelastic*</u>. What percentage of the their total initial kinetic energy will be <u>lost</u> in the collision?

(a) 75%

$\boxed{\text{(b) 50\%}}$

~~(c) 0%~~

(d) 25%

~~(e) 100%~~

(Note that "0%" and "100%" are rather absurd answers)

$\boxed{K_i = \frac{1}{2}mv_o^2}$

→ Perfectly inelastic $\vec{P}_i = \vec{P}_f$ with $\vec{V}_{1f} = \vec{V}_{2f} = \vec{V}_f$

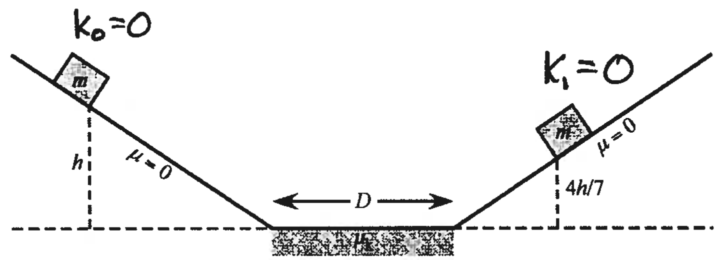
$\langle +mv_o \rangle = (2m)\vec{V}_f \rightarrow \boxed{V_f = V_o/2}$

Then $K_f = \frac{1}{2}(2m)\left(\frac{V_o}{2}\right)^2 = \frac{1}{4}mV_o^2$

or: $K_f = \frac{1}{2}K_i \Rightarrow$ $\boxed{\text{50\% of initial KE was lost}}$

The following problem will be hand-graded. <u>*Show all your work for this problem.*</u> *Make no marks and leave no space on your answer card for it.*

[III] A block is placed on a frictionless ramp at a height h, and released from rest. It slides down the ramp to a rough horizontal surface, sliding a distance D along that surface to a second, upward-directed frictionless ramp. It is observed to rise to a height (4/7) h before coming to a momentary stop. It then starts back down the ramp and recrosses the rough surface.



(15 points) If the length of the rough surface is $D = 1.5\ h$, what is the coefficient of friction for that surface? Express your answer as a numerical value, to two-digit precision.

$W_{diss} = \Delta K + \Delta U$, but $\Delta K = 0$ (block starts, ends at rest)

↳ on horizontal surface, $f_K = \mu_K N = \mu_K mg$ [because $a_y \equiv 0$]

so $W_{diss} = -f_K D = -\mu_K mg D = -\frac{3}{2}\mu_K mgh$

then, with $U_1 = \frac{4}{7}mgh$ and $U_0 = mgh$, we have:

$$W_{diss} = \Delta U \longrightarrow -\frac{3}{2}\mu_K mgh = \frac{4}{7}mgh - \frac{7}{7}mgh = -\frac{3}{7}mgh$$

$$\frac{3}{2}\mu_K = \frac{3}{7}$$

$$\boxed{\mu_K = \frac{2}{7} = 0.29}$$

Question value 5 points — mark your answer in space #2 on the answer card.

(2) Where along the rough horizontal surface will the block eventually come to a complete stop?

(a) at a distance $D/4$ from the left side.

(b) at a distance $D/2$ from the left side.

(c) at a distance $3D/4$ from the left side.

(d) at a distance $2D/3$ from the left side.

(e) at a distance $D/3$ from the left side.

energy loss for <u>one</u> full crossing of rough surface is

$$W_{diss} = -\frac{3}{7}mgh = -\frac{3}{7}E_i$$

→ after <u>re-crossing</u> surface, right-to-left, block will have <u>Mechanical Energy</u>

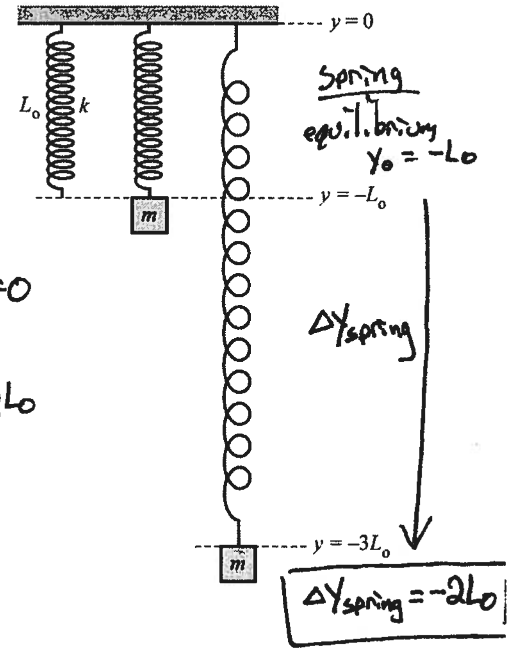
$$E_2 = E_i - \frac{3}{7}E_i - \frac{3}{7}E_i = \boxed{\frac{1}{7}E_i}$$

⇒ To lose <u>remaining</u> energy after recrossing, block must slide <u>one</u> third of the way across

(if you lose $\frac{3}{7}E_i$ in distance D, then you lose $\frac{1}{7}E_i$ in distance $D/3$)

→ $\boxed{\text{stops at } D/3 \text{ from the left side}}$

An ideal massless spring hangs from the ceiling. When nothing is attached to the spring, its natural length is L_o, so that the lower end hangs at position $y_o = -L_o$, relative to the ceiling. A mass m is then attached to the lower end of the spring, and released from rest. The mass drops to a position $y_1 = -3L_o$, where it stops mometarily before bouncing back upward.



Question value 5 points

(3) What is the elastic constant for the spring?

(a) $k = mg/3L_o$

(b) $\boxed{k = mg/L_o}$

(c) $k = 2mg/L_o$

(d) $k = 3mg/L_o$

(e) $k = mg/2L_o$

Block begins and ends at rest: $K_i = K_f = 0$

• initial PE: $U_s = 0$, $U_g = -mgL_o$

• final PE: $U_s = \frac{1}{2}k(2L_o)^2$ $U_g = -3mgL_o$

$E_i = E_f \Rightarrow -mgL_o = \frac{1}{2}k(4L_o^2) - 3mgL_o$

$\cancel{2}mg\cancel{L_o} = \cancel{2}kL_o^{\cancel{2}}$

$$\boxed{k = \frac{mg}{L_o}}$$

(4) What will be the position of the block (relative to the ceiling) at the moment it is moving back upward, with maximum speed?

(a) $y = -1.5L_o$

(b) $y = -2.5L_o$

(c) $y = -1.0L_o$

(d) $\boxed{y = -2.0L_o}$

(e) $y = -3.0L_o$

Maximum speed implies zero acceleration (neither gaining, nor losing, velocity)

→ at this moment, net force on block is zero

ie block is in (momentary) equilibrium

F_s $(a=0)$ mg

$\langle +F_s \rangle + \langle -mg \rangle = 0$

$\langle -k(Y - Y_o) \rangle + \langle -mg \rangle = 0$

$-\left(\frac{mg}{L_o}\right)(Y - Y_o) = mg$

$Y - Y_o = -L_o$ →

$Y = -L_o + Y_o$

$= -L_o + (-L_o)$

$\boxed{Y = -2L_o}$

(5) Suppose that, instead of simpy releasing the block, it is gently lowered until it hangs at rest, with the block in equilibrium. What will be the position of the block at this moment?

(a) $y = -3.0L_o$

(b) $\boxed{y = -2.0L_o}$

(c) $y = -1.0L_o$

(d) $y = -1.5L_o$

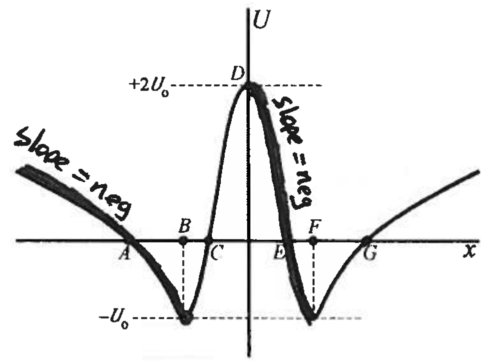
(e) $y = -2.5L_o$

Hanging at rest: block is in equilibrium

⇒ see prior problem: math is the same!

$$\boxed{Y = -2L_o}$$

George P. Burdell has engineered a 21st-century elastic device called the "sproing"; unlike an antiquated 20th-century spring, the sproing has a potential energy function given by the graph at right.



Question value 5 points

(6) In what intervals will the sproing exert a positively-directed force?

(a) between A and C, and also between E and G.

(b) between B and D, and also to the right of F.] wrong sign for slope!

(c) nowhere in the regions shown.

$\boxed{\text{(d) to the left of B, and also between D and F.}}$

(e) to the left of A, between C and E, and to the right of G.

Force is found as gradient of potential energy $\vec{F}_x = \langle -\frac{dU}{dx} \rangle$

$\Rightarrow$ negative of slope of $U(x)$

so: positive force is found $\boxed{\text{when } U(x) \text{ has negative slope}}$

Question value 5 points

(7) A mass is attached to the sproing when it is at position B, and the "system" is defined to be the mass and sproing together. The block is moved from position B to position G. How much work was done on the block by the sproing?

(a) $W_s = +2U_o$

(b) $W_s = -3U_o$

(c) $\boxed{W_s = -U_o}$

(d) $W_s = -2U_o$

(e) $W_s = +U_o$

definition of Work–PE relationship, for conservative forces is $\Delta U \equiv -W_{cons}$

[so that pos work removes PE, to add to KE and neg work adds to PE, by removing from KE]

$$W_{sproing} = -\Delta U = -(U_f - U_i) = -(U_g - U_b) = -(0-(-U_0))$$

$$\boxed{W_s = -U_o}$$

Question value 5 points

(8) Suppose the block began at rest at position B, and was observed to have kinetic energy K as it passed through position G. How much external work was done on the system, as the block was moved from B to G?

(a) $W_{ext} = +2U_o - K$

(b) $\boxed{W_{ext} = +U_o + K}$

(c) $W_{ext} = -U_o + K$

(d) $W_{ext} = +U_o - K$

(e) $W_{ext} = -2U_o + K$

① $W_{TOT} = \Delta K \Rightarrow$ Apply Work-KE theorem

$W_{ext} + W_s = K - 0$

$W_{ext} + (-U_o) = K \longrightarrow \boxed{W_{ext} = K + U_o}$

OR:

② Apply general work-energy theorem: $W_{ext} + \cancel{W_{diss}}^{\to 0} = \Delta K + \Delta U$

$W_{ext} = (K-0) + (0-(-U_o))$

$\boxed{W_{ext} = K + U_o}$

Two sprinters run a footrace of total distance D. Both start from rest. Francine runs with constant applied force F_O, for the entire duration of the race. Peter runs with constant power output P_O, for the entire duration of the race.

Question value 5 points

(9) Suppose that Francine crosses the finish line with a final speed v_f. Let W_1 be the amount of work required for her to go from rest to speed $v_f/2$, which occurs over a total distance D_1. Let W_2 be the amount of work required for her to go from speed $v_f/2$ to speed v_f, which occurs in a distance D_2. How does the distance D_2 compare to the distance D_1?

(a) $D_2 = D_1$

(b) $D_2 = 2D_1$

(c) $D_2 = D_1/2$

$\boxed{\text{(d) } D_2 = 3D_1}$

(e) $D_2 = 4D_1$

Final KE : $K_f = \frac{1}{2} m v_f^2$

KE between the two legs: $K_b = \frac{1}{2} m (v_f/2)^2 = \frac{1}{8} m v_f^2 = \frac{1}{4} K_f$

- Work on leg 1 : $W_1 = \Delta K = \frac{1}{4} K_f - 0 = \frac{1}{4} K_f$
- Work on leg 2 : $W_2 = \Delta K = K_f - \frac{1}{4} K_f = \frac{3}{4} K_f$

$\boxed{W_2 = 3W_1}$

Now : Work by constant force in 1D is $W = \vec{F} \cdot \Delta \vec{x} = F_o \cdot D$

→ for constant force, we see

$$\frac{W_1}{D_1} = F_o = \frac{W_2}{D_2} \rightarrow D_2 = \frac{W_2}{W_1} D_1 = \frac{3W_1}{W_1} D_1 \Rightarrow \boxed{D_2 = 3D_1}$$

Question value 5 points

(10) Suppose that Peter crosses the finish line at time t. If his speed at time $t/2$ is v, what is his speed at the end of the race?

$\boxed{\text{(a) } 1.41\, v}$

(b) $1.73\, v$

(c) $2.00\, v$

(d) $2.23\, v$

(e) $1.00\, v$

Consider full race at constant power P_o :

$$\Delta E = P_o \Delta t \rightarrow E_f - \cancel{E_i} = P_o (t_f - \cancel{t_i})$$

$$\frac{1}{2} m v_f^2 = P_o t_f$$

$$v_f = \sqrt{\frac{2P_o}{m}}\, t_f^{1/2}$$

The important thing to carry away from this is:

$v \sim t^{1/2}$ (all other parameters are constants)

we can thus write

$$\frac{v_f}{v_{half}} = \frac{t_f^{1/2}}{t_{half}^{1/2}} \Rightarrow \frac{v_f}{v} = \frac{t^{1/2}}{(t/2)^{1/2}} = \sqrt{2}\, \frac{t^{1/2}}{t^{1/2}} = \sqrt{2}$$

$$\boxed{v_f = \sqrt{2}\, v = 1.414\, v}$$