Physics 2211 B/C
Spring 2016
Exam 1

Test form 161

Name Solutions

Recitation Section (see back of test):__________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number **(ABOVE)** in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

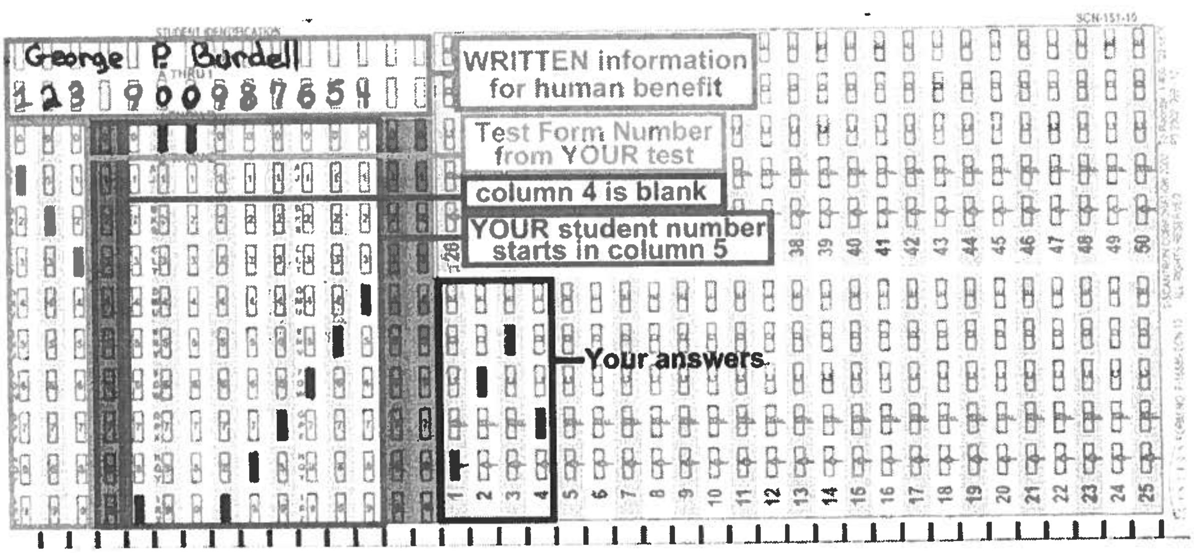
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

Your test form is: 161



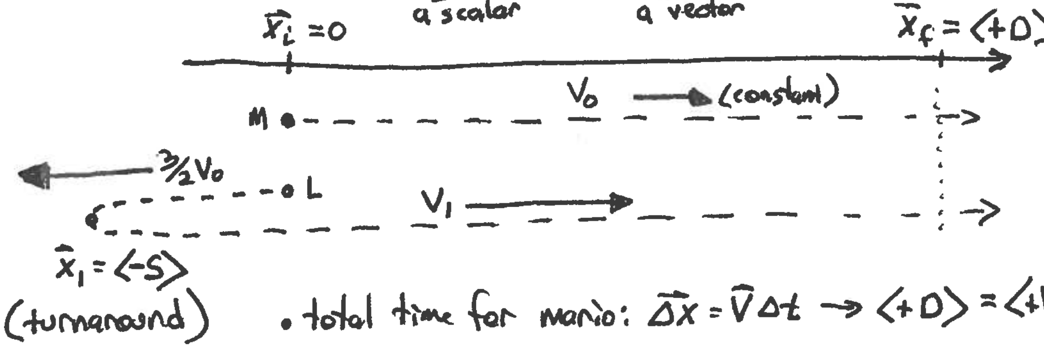
Our next test will be on Monday, February 22!

The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[I] *(20 points)* Super Mario and his brother Luigi are in a footrace, covering a total distance D. Mario sets out with constant speed v_0, heading straight for the finish line. Luigi sets out at a constant speed $\frac{3}{2}v_0$, but in his excitement, he runs in a direction directly opposite to the finish line. At the point when Mario is 2/5 of the way to the finish line, Luigi realizes his mistake, and turns around to run at a constant $\boxed{\text{speed}}$ v_1 toward the finish line. (Assume neither runner experiences any power-ups.)

Find the minimum value for v_1 that will allow Luigi to catch up to his brother Mario and end the race in a tie. Then, determine Luigi's average $\boxed{\text{speed}}$ and average $\boxed{\text{velocity}}$ for the race. In each case, express your answer in terms of v_0.

a scalar   a vector



let positive direction be: "from start line to finish line"

- total time for mario: $\vec{\Delta x} = \vec{V}\Delta t \rightarrow \langle +D \rangle = \langle +V_0 \rangle \Delta t_{TOT} \rightarrow \Delta t_{TOT} = D/V_0$
- "wrong-way time" for luigi: $\vec{\Delta X}_M = 2/5 D \rightarrow$ requires time $\Delta t = \frac{2}{5}\Delta t_{TOT} = \frac{2}{5}\frac{D}{V_0}$

so wrong-way distance is $\vec{\Delta X}_L = \vec{V}_L \Delta t \Rightarrow \langle -S \rangle = \langle -3/2 V_0 \rangle \cdot \frac{2D}{5V_0}$

$S = 3/5 D$

so: in remaining time, $\Delta t_2 = 3/5 \Delta t_{TOT} = \frac{3D}{5V_0}$, luigi must travel distance $S + D = 8/5 D$

$\Rightarrow \vec{\Delta X}_1 = \vec{V}_1 \Delta t_1 \rightarrow \langle +8/5 D \rangle = \langle +V_1 \rangle \cdot \frac{3D}{5V_0} \rightarrow \boxed{V_1 = 8/3 V_0}$ speed is a scalar answer

Now, on to average velocity (a vector) $\vec{V}_{avg} = \frac{\vec{\Delta X}_{TOT}}{\Delta t_{TOT}}$

$\rightarrow$ Since Mario + luigi experience same displacement, $\langle +D \rangle$ in same time, D/V_0, they must have same average velocity

obviously, $\vec{V}_{m,avg} = \langle +V_0 \rangle$, so $\boxed{\vec{V}_{L,avg} = \langle +V_0 \rangle}$ vector answer!

Finally, average speed is $V_{av} = \frac{\text{total distance}}{\text{total time}} = \frac{|-S| + |S+D|}{D/V_0} = \frac{11/5 D}{D/V_0}$

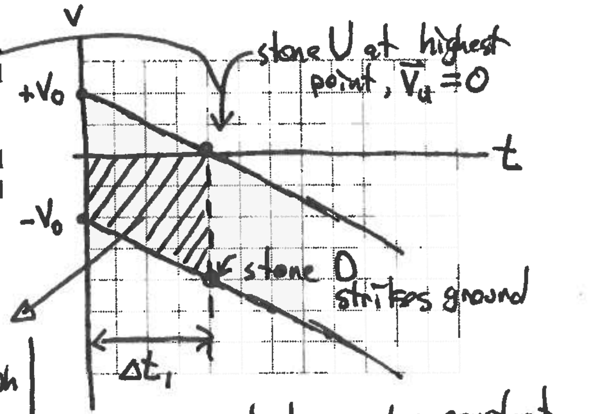
$$\boxed{V_{av} = \frac{11}{5} V_0}$$ scalar answer

The following problem will be hand-graded. *__Show all your work for this problem__. Make no marks and leave no space on your answer card for it.*

[II] *(20 points)* Uma and Deborah are standing at the top of a cliff of height $H = 30.0$ m. Each of them throws a stone with an unknown initial speed v_0. Umma throws her stone straight upward, while Deborah throws her stone straight downward. They note that Deborah's stone strikes the ground at the exact same instant that Uma's stone reaches its highest point.

On the grid at right, graph the velocities of the two stones as functions of time. Indicate on the graph the moment Uma's stone reaches its highest point, and explain how the height H of the building is represented in the graph.

Determine the speed with which the two stones were initially thrown. It will be advantageous to work the problem algebraically, and save your numerical substitutions until the final step.



(negative) displacement is found as area under curve, so

$H = |\text{area shaded in graph}|$

constant accel = constant slope

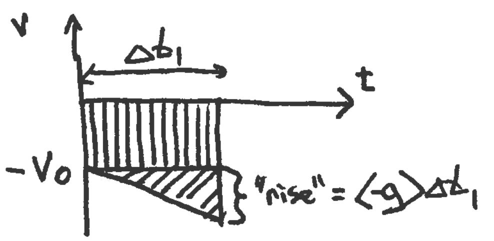
→ same negative slope for each

• looking at plot for stone U:

$\Delta \vec{V}_u = \langle -g \rangle \Delta t \rightarrow \langle 0 \rangle - \langle +V_0 \rangle = \langle -g \rangle \Delta t_1$

$\rightarrow \Delta t_1 = \dfrac{V_0}{g}$ = rise time for stone U AND fall time for stone S

• looking at plot for stone D: area "under" curve (which is negative) gives the displacement (which is also negative)



$\langle -H \rangle = -(\text{rectangle area}) + \left(-(\text{triangle area})\right)$

$= (-V_0)\Delta t_1 + \frac{1}{2}(-g\Delta t_1)(\Delta t_1)$

$H = V_0 t_1 + \frac{1}{2} g \Delta t_1^2$ where $\Delta t_1 = \dfrac{V_0}{g}$

$H = V_0\left(\dfrac{V_0}{g}\right) + \frac{1}{2} g \left(\dfrac{V_0}{g}\right)^2 = \dfrac{3}{2}\dfrac{V_0^2}{g}$

$\rightarrow \boxed{V_0 = \sqrt{\frac{2}{3} g H}}$

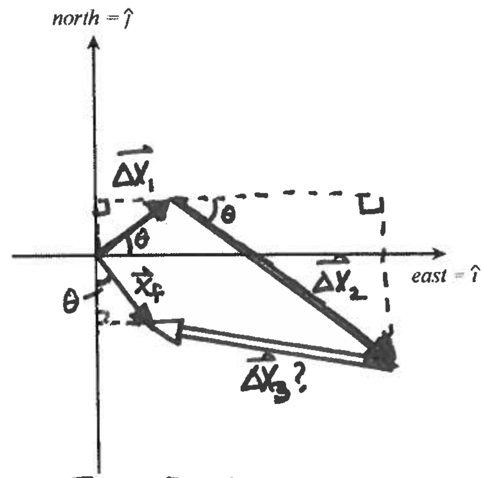
The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

[III] *(20 points)* While on a camping trip, you hike a distance d from your campsite, in a direction 36.9° north of east. You then hike a distance $3d$ in a direction 36.9° south of east. At that location, you trip over a branch and bang your bead on a rock. When you regain your senses, you find that you are somewhere else—apparently you have wandered aimlessly while mentally blacked out. Pulling out the GPS on your phone, you discover that you are now located a distance d from the campsite, in a direction 36.9° east of south What displacement did you make while blacked out?

Express your answer in component form using $\hat{\imath}$ and $\hat{\jmath}$, and as a magnitude and direction relative to some compass heading.

For this problem, the following trig relations will be useful:

$$\sin(36.9°) = \tfrac{3}{5} \qquad \cos(36.9°) = \tfrac{4}{5}$$



Known displacements:

$$\vec{\Delta X_1} = \langle +d\cos\theta \rangle \hat{\imath} + \langle +d\sin\theta \rangle \hat{\jmath}$$
$$= \langle +\tfrac{4}{5}d \rangle \hat{\imath} + \langle +\tfrac{3}{5}d \rangle \hat{\jmath}$$

$$\vec{\Delta X_2} = \langle +3d\cos\theta \rangle \hat{\imath} + \langle -3d\sin\theta \rangle \hat{\jmath}$$
$$= \langle +\tfrac{12}{5}d \rangle \hat{\imath} + \langle -\tfrac{9}{5}d \rangle \hat{\jmath}$$

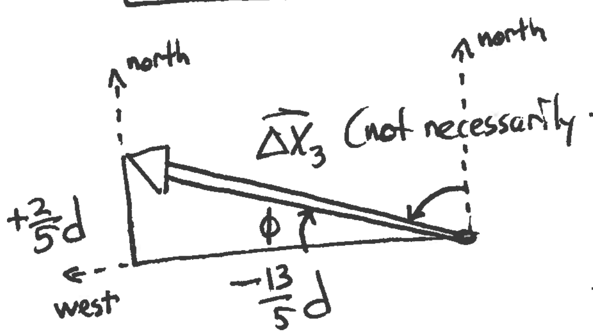
Net displacement $\vec{\Delta X_{TOT}} = \vec{X_f} - \vec{X_i} = \left[\langle +\tfrac{3}{5}d \rangle \hat{\imath} + \langle -\tfrac{4}{5}d \rangle \hat{\jmath}\right] - [\langle 0 \rangle]$

but $\vec{\Delta X_{TOT}} = \vec{\Delta X_1} + \vec{\Delta X_2} + \vec{\Delta X_3}$

so, $\vec{\Delta X_3} = \vec{\Delta X_{TOT}} - \vec{\Delta X_1} - \vec{\Delta X_2}$

$$= \left[\langle +\tfrac{3}{5}d \rangle \hat{\imath} + \langle -\tfrac{4}{5}d \rangle \hat{\jmath}\right] - \left[\langle +\tfrac{4}{5}d \rangle \hat{\imath} + \langle +\tfrac{3}{5}d \rangle \hat{\jmath}\right] - \left[\langle +\tfrac{12}{5}d \rangle \hat{\imath} + \langle -\tfrac{9}{5}d \rangle \hat{\jmath}\right]$$

$$\boxed{\vec{\Delta X_3} = \langle -\tfrac{13}{5}d \rangle \hat{\imath} + \langle +\tfrac{2}{5}d \rangle \hat{\jmath}}$$



$$|\vec{\Delta X_3}| = \sqrt{\left(\tfrac{2}{5}d\right)^2 + \left(\tfrac{13}{5}d\right)^2} = \boxed{\frac{\sqrt{173}}{5}d} \approx 2.63d$$

$$\tan\phi = \frac{|\tfrac{2}{5}d|}{|\tfrac{13}{5}d|} = \frac{2}{13} \rightarrow \phi = 8.75°$$

so direction is $\boxed{8.75° \text{ North of West}}$

[or 81.3° west of north]

Question value 5 points

(1) In a free-fall problem, H represents a height, and g represents the magnitude of gravitational acceleration. Which of the the following algebraic expressions might possibly be an expression for *elapsed time*?

Dimensional analysis:

(a) $\Delta t = \sqrt{2gH}$

(b) ~~$\Delta t = \sqrt{2H} - \sqrt{3g}$~~ → first off, this is nonsense – you cannot subtract $\sqrt{g}$ from $\sqrt{H}$ because the units are obviously inequivalent

(c) $\Delta t = \sqrt{g/2H}$

(d) $\boxed{\Delta t = \sqrt{H/3g}}$ → what about the others? Well, ignore factors of 2 or 3 (they don't change the units)

(e) $\Delta t = \sqrt{3/gH}$

$\sqrt{gH} \to \left[\frac{\text{length}}{\text{time}^2}\cdot \text{Length}\right]^{1/2} = \left[\frac{\text{length}}{\text{time}}\right]$ NOPE!

$\boxed{\sqrt{H/g} = \left[\text{length}\cdot \frac{\text{Time}^2}{\text{length}}\right]^{1/2} = [\text{Time}]}$ Yes!!

$\sqrt{g/H} \to \left[\frac{\text{length}}{\text{time}^2}\cdot \frac{1}{\text{length}}\right]^{1/2} = \left[\frac{1}{\text{time}}\right]$ Nope!

$\frac{1}{\sqrt{gH}} = \left[\frac{\text{Time}}{\text{length}}\right]$ Nope!

Question value 5 points

(2) A rolling cart is given an intital shove up an inclined ramp. It travels a distance D up the ramp to a stop, and then back down to its starting position. During what parts of its motion is the cart accelerating?

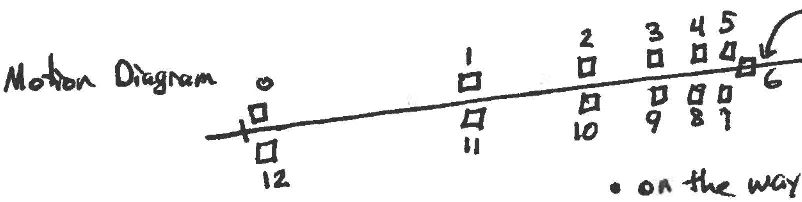
(a) While it is on the way up, at the moment it is stationary at the top, and while it is on the way down.

(b) Only while on the way down.

(c) While it is on the way up and while it is on the way down, but not at the moment it is stationary at the top.

(d) Only while on the way up.

(e) Only at the moment it is stationary at the top.



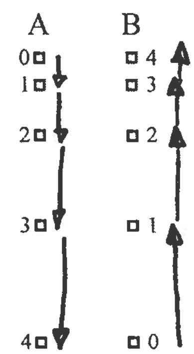
• rising, with speed ↓: downslope accel

• at stationary moment: changing from $\vec{v}$=up to $\vec{v}$=down = downslope accel

• on the way down, with speed ↑ = downslope accel

Question value 5 points

(3) The figure at right depicts motion diagrams for two moving objects, A and B. Which of the statements below best characterizes the two diagrams?



(a) The velocities of A and B have the ~~same sign~~; their accelerations have the ~~opposite~~ sign.

(b) The velocities of A and B have opposite signs; their accelerations have ~~opposite~~ signs.

(c) We cannot compare the signs of $\vec{v}$ or $\vec{a}$, without knowing the coordinate system being used.

(d) The velocities of A and B have opposite signs; their accelerations have the same sign.

(e) The velocities of A and B have the ~~same sign~~; their accelerations have the same sign.

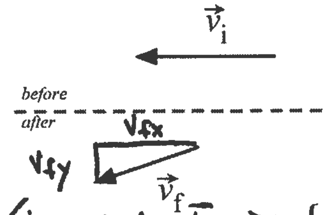
A: $\vec{v}$ downward, increasing in magnitude: $\vec{a}$ = downward

B: $\vec{v}$ upward, increasing in magnitude: $\vec{a}$ = downward

→ $\vec{V}_A$ and $\vec{V}_B$ have opposite signs
$\vec{a}_A$ and $\vec{a}_B$ have the same sign

Question value 5 points

(4) At a particular moment in time, a car has the initial velocity vector shown at top right. A few moments later, its velocity is as shown at bottom right. Which of the vectors below best depicts the average acceleration of the car during this interval?



(a) ↗

(b) ↖

(c) →

(d) ↙

$\boxed{\text{(e) ↘}}$

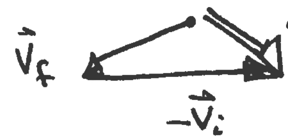
$\vec{a} = \frac{\vec{\Delta v}}{\Delta t}$

→ direction of $\vec{a}$ is the same as the direction of $\vec{\Delta V} = \vec{V}_f - \vec{V}_i$

$= \vec{V}_f + (-\vec{V}_i)$

(Note that $\vec{V}_{f,x}$ is clearly less than $\vec{V}_i$ ⇒ $\vec{a}_x$ is rightward)

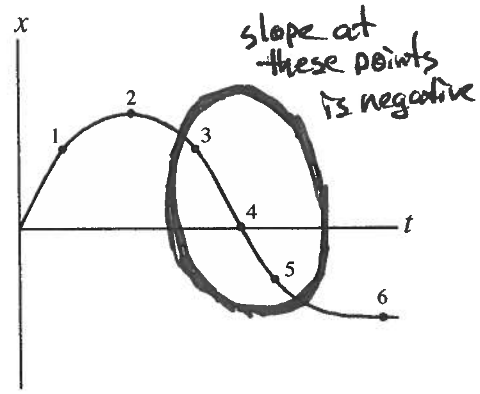
Graphically:



$\vec{\Delta V}$ and $\vec{a}$ are "down and right"

Question value 5 points

(5) The figure at right shows a position-*vs*-time graph for an object moving along a horizontal axis, with *rightward* being the positive direction. At which of the numbered points is the object moving *to the left?*



slope at these points is negative

(a) At points 3, 4, and 5.
(b) At points 5 and 6.
(c) At *none* of the indicated points.
(d) At points 1, 2, and 3.
(e) At points 1, 3, and 5.

Velocity is negative

BUT:

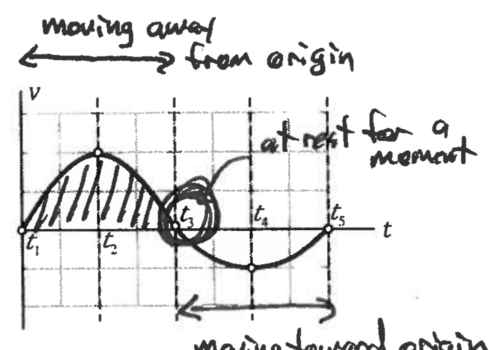
on x-vs-t graph, velocity is slope

→ where is slope = negative

$\boxed{\text{At } t_3, t_4, t_5}$

Question value 5 points

(6) The graph at right displays the velocity of a particle that starts at the origin. Which of the indicated points in time represents the particle being furthest from the origin?



(a) At time t_4.

$\boxed{\text{(b) At time } t_3.}$

(c) At time t_2.

(d) At time t_5.

~~(e) At time t_1.~~

obviously, at t_1 the particle is at the origin!

recall that area under curve gives displacement:

$\vec{\Delta X} = \vec{X}_f - \vec{X}_i$ zero, starts at origin

Furthest from origin: $|\vec{\Delta X}|$ = maximum

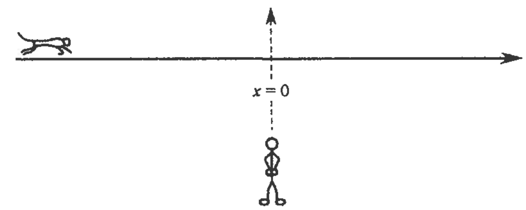
→ max positive area $\boxed{\text{at time } t_3}$

The next two questions involve the following situation:

A photo-safari guide sees a cheetah run from his left to his right, in a straight line. Using a coordinate system with the origin straight in front of the guide, and rightward as the positive direction, the cheetah's velocity is given by the expression:

$$\vec{v}(t) = \vec{A}t^2 + \vec{B}$$

where $\vec{A}$ and $\vec{B}$ are have the values $\vec{A} = \langle +15\ \mathrm{m/s^3} \rangle$ and $\vec{B} = \langle -5\ \mathrm{m/s} \rangle$. (Note that with these values, the cheetah is actually travelling *to the left* at $t = 0$, but it reverses direction very soon after that.)



Question value 5 points

(7) If the cheetah passes directly in front of the guide ($\vec{x} = 0$) at time $\boxed{t = 2 \text{ seconds}}$, where was the cheetah at time $t = 0$?

call this time "T"

(a) $\vec{x}_0 = \langle -90\ \mathrm{m} \rangle$

(b) $\vec{x}_0 = \langle -110\ \mathrm{m} \rangle$

(c) $\vec{x}_0 = \langle -50\ \mathrm{m} \rangle$

(d) $\vec{x}_0 = \langle -70\ \mathrm{m} \rangle$

(e) $\boxed{\vec{x}_0 = \langle -30\ \mathrm{m} \rangle}$

$\vec{x}$ is found as antiderivative (ie integral) of $\vec{V}(t)$

$\vec{X}(t) = \int \vec{V}(t)\,dt + \vec{C} = \frac{1}{3}\vec{A}t^3 + \vec{B}t + \vec{C}$ where $\vec{C}$ = unknown

$\vec{X}(T) = 0$ lets us determine $\vec{C}$: $0 = \frac{1}{3}\vec{A}T^3 + \vec{B}T + \vec{C}$

$\vec{C} = -\frac{1}{3}\vec{A}T^3 - \vec{B}T = \langle -40\ m \rangle - \langle -10\ m \rangle = \langle -30\ M \rangle$

Then $X(0) = \frac{1}{3}\vec{A}\cdot(0)^3 + \vec{B}(0) + \vec{C} \rightarrow \boxed{\vec{X}(0) = \vec{C} = \langle -30\ m \rangle}$

Question value 5 points

(8) What was the average acceleration of the cheetah over the interval $0 \le t \le 2$ seconds?

(a) $\vec{a}_{av} = \langle +20\ \mathrm{m/s^2} \rangle$

(b) $\vec{a}_{av} = \langle +40\ \mathrm{m/s^2} \rangle$

(c) $\boxed{\vec{a}_{av} = \langle +30\ \mathrm{m/s^2} \rangle}$

(d) $\vec{a}_{av} = \langle +60\ \mathrm{m/s^2} \rangle$

(e) $\vec{a}_{av} = \langle +50\ \mathrm{m/s^2} \rangle$

$\vec{a}_{av}$ is defined as $\vec{a}_{av} = \frac{\Delta \vec{V}}{\Delta t} = \frac{\vec{V}_f - \vec{V}_i}{t_f - t_i}$

so: $\vec{a}_{av} = \frac{\vec{V}(T) - \vec{V}(0)}{T - 0}$

$= \frac{(\vec{A}T^2 + \vec{B}) - (0 + \vec{B})}{T} = \frac{\vec{A}T^2}{T}$

$$\boxed{\vec{a}_{av} = \vec{A}T = \langle +30\ m/s^2 \rangle}$$