Physics 2211M Test form 151 Name Solutions

Summer 2016

Exam 1 Recitation Section (see back of test):________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

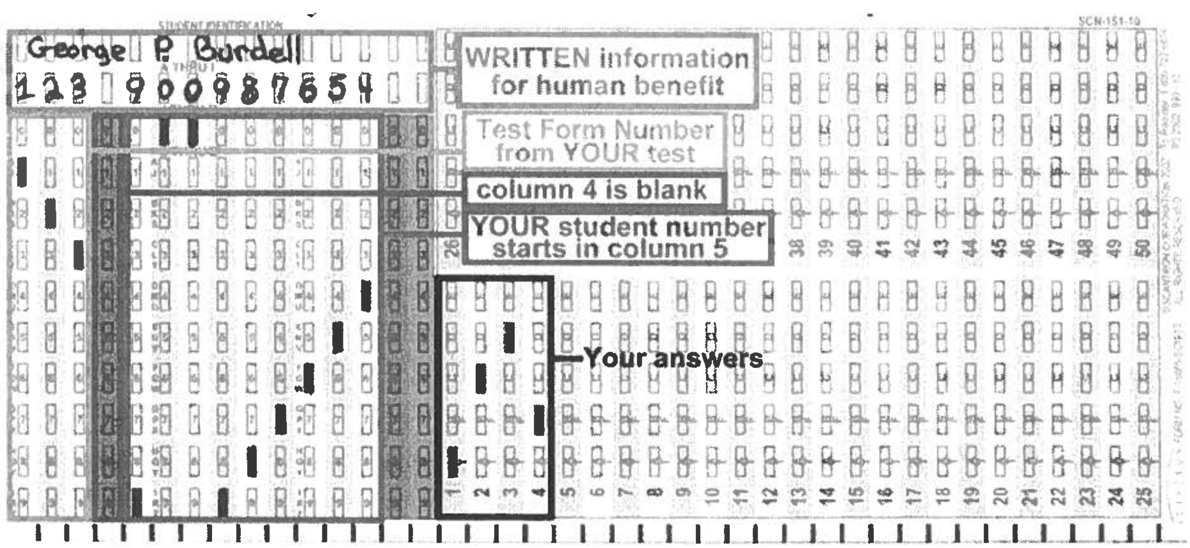
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

Your test form is: 151



Our next test will be on Wednesday, June 29!

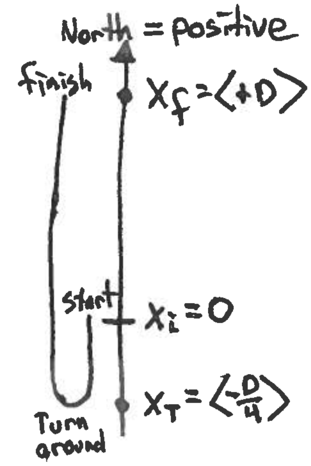
The following problem will be hand-graded. Show all your work for this problem. *Make no marks and leave no space on your answer card for it.*

[I] *(20 points)* Buzz is in a footrace that covers a straight-line distance D, running from south to north. Being highly distractable, Buzz starts off running the wrong way (south) with a speed v for a distance $D/4$, after which he turns around and runs the right way (north) with a speed $3v$ to finish the race.

(i) What is Buzz's average velocity for the entire race?

(ii) What is Buzz's average speed for the entire race?

[Hint for both questions: start by finding the total elapsed time for Buzz to finish the race…]



- time spent travelling wrong way:

$$\vec{v} = \frac{\Delta x}{\Delta t} \rightarrow \langle -v \rangle = \frac{\langle -D/4 \rangle}{\Delta t_1}$$

$$\boxed{\Delta t_1 = D/4v} = \frac{3D}{12v}$$

- time spent traveling right way:

$$\vec{v} = \frac{\Delta x}{\Delta t} \rightarrow \langle +3v \rangle = \frac{\langle +D \rangle - \langle -D/4 \rangle}{\Delta t_2}$$

$$\boxed{\Delta t_2 = \frac{5D/4}{3v} = \frac{5D}{12v}}$$

"$\frac{D}{v}$" has units $\frac{[\text{length}]}{[\text{length}/\text{time}]} = [\text{time}]$

→ expression has proper units for time Δt

So:

(i) average velocity: $\vec{v}_{av} = \frac{\overrightarrow{\Delta x_{TOT}}}{\Delta t_{TOT}} = \frac{\vec{x}_f - \vec{x}_i}{\Delta t_1 + \Delta t_2} = \frac{\langle +D \rangle - \langle 0 \rangle}{\frac{3D}{12v} + \frac{5D}{12v}}$

requires a vector answer

$$\vec{v}_{av} = \frac{\langle +D \rangle}{8D/12v} = \langle + \frac{12v}{8D} D \rangle = \boxed{\langle +\tfrac{3}{2} v \rangle}$$

positive: northward

(ii) average speed $v_{av} = \frac{\text{Total distance}}{\text{total time}}$

$$= \frac{D/4 + 5D/4}{8D/12v} = \frac{12v}{8D} \cdot \frac{6D}{4} = \boxed{\frac{9}{4} v}$$

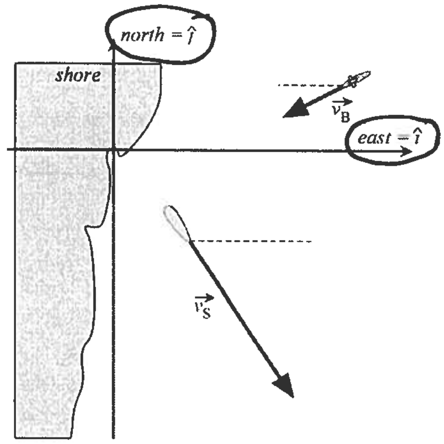
speed is a scalar answer (no direction)

[Note that both answers obviously have speed/velocity units…]

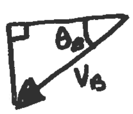
The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[II] *(20 points)* Buzz is piloting the Submersible Wreck off the coast of Georgia. His GPS receiver indicates that, relative to shore, he is travelling with a speed v in a direction 27.0° south of west. A ship is heading out to sea, moving relative to shore with a speed $3.00\,v$ in a direction 56.0° south of east.

What will the Wreck's sonar indicate as the the velocity of the ship relative to Buzz? Express your answer in both Cartesian component form and as a magnitude/direction pair.

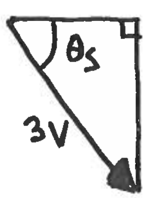


Both velocities are relative to shore/Earth



$$\vec{V}_{BE} = \langle -V\cos 27^\circ \rangle \hat{\imath} + \langle -V\sin 27^\circ \rangle \hat{\jmath} = \langle -0.891V \rangle \hat{\imath} + \langle -0.454V \rangle \hat{\jmath}$$

Buzz to Earth

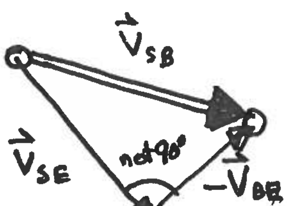


$$\vec{V}_{SE} = \langle +3V\cos 56^\circ \rangle \hat{\imath} + \langle -3V\sin 56^\circ \rangle \hat{\jmath} = \langle +1.678V \rangle \hat{\imath} + \langle -2.487V \rangle \hat{\jmath}$$

ship to Earth

So — relative velocity statement is: $\vec{V}_{SB} + \vec{V}_{BE} = \vec{V}_{SE}$

$$\rightarrow \vec{V}_{SB} = \vec{V}_{SE} - \vec{V}_{BE} = \vec{V}_{SE} + (-\vec{V}_{BE})$$



subbing components:

$$\vec{V}_{SB} = \langle +1.678V \rangle \hat{\imath} + \langle -2.487V \rangle \hat{\jmath} - \langle -0.891V \rangle \hat{\imath} - \langle -0.454V \rangle \hat{\jmath}$$

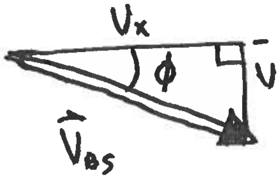
["lots" of East, and "a little" South]

$$\boxed{\vec{V}_{SB} = \langle +2.57V \rangle \hat{\imath} + \langle -2.03V \rangle \hat{\jmath}}$$

3-digit precision in answer

matches rough graphical estimation

then $|\vec{V}_{SB}| = \sqrt{V_x^2 + V_y^2} = \boxed{3.28V}$ (note that symbol V has implicit speed units, so we do not need to add our own)

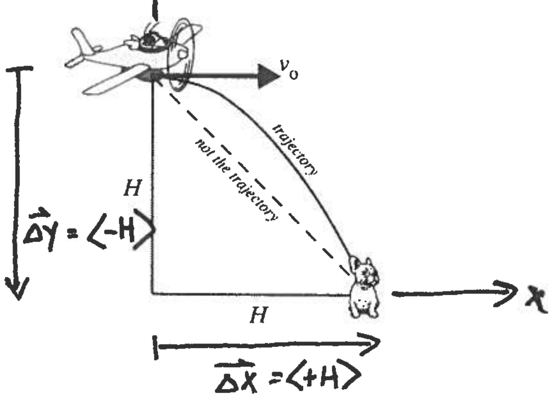


direction: $\phi = \tan^{-1}\left(\frac{|V_y|}{|V_x|}\right) = 38.4^\circ$

so $\boxed{\phi = 38.4^\circ \text{ south of East}}$

The following problem will be hand-graded. *__Show all your work for this problem__.* *Make no marks and leave no space on your answer card for it.*

[III] *(20 points)* Buzz is flying a small plane, with plans to drop a gold paint-bomb on Uga X. (He chose not to use a stink-bomb because he figured no one would notice the difference.) He is in level flight at a vertical distance *H* above the ground, and releases the bomb when he is at a horizontal distance *H* in front of Uga. (Yes, it is the <u>same</u> *H* for both the vertical and horizontal distances.)



Find an expression for the *specific* speed v_0 that the plane must have, in order for the bomb to strike precisely on target. Express your answer symbolically in terms of *H* and *g*.

① Vertical motion = free fall from height $H = y_i$ with $V_{yi} \equiv 0$

$$\vec{\Delta y} = \vec{V}_{iy}\Delta t + \frac{1}{2}\langle -g\rangle \Delta t^2 \rightarrow \langle -H\rangle = 0 + \frac{1}{2}\langle -g\rangle \Delta t^2$$

$$\Delta t = \sqrt{\frac{2H}{g}} \quad \left[\sqrt{\frac{\text{length}}{\text{length}/\text{time}^2}} = \text{time}\right] \rightarrow \text{valid dimensions}$$

② horizontal motion = constant velocity: $\vec{V}_x = \langle +V_0\rangle = \text{constant}$

so $\vec{\Delta x} = \vec{V}_x \Delta t$

$$\langle +H\rangle = \langle +V_0\rangle \Delta t \Rightarrow H = V_0\left[\sqrt{\frac{2H}{g}}\right]$$

$$V_0 = H\sqrt{\frac{g}{2H}} = \boxed{\sqrt{\frac{gH}{2}}}$$

$g: \frac{[\text{length}]}{[\text{time}]^2}$

$H: [\text{length}]$

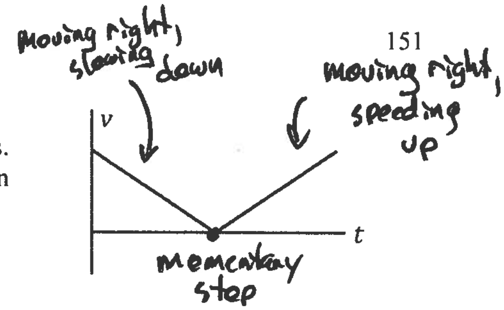
$gH: \frac{[\text{length}]^2}{[\text{time}]^2}$

$\sqrt{\frac{gH}{2}}: \sqrt{\frac{\text{length}^2}{\text{time}^2}} = \left[\frac{\text{length}}{\text{time}}\right]$

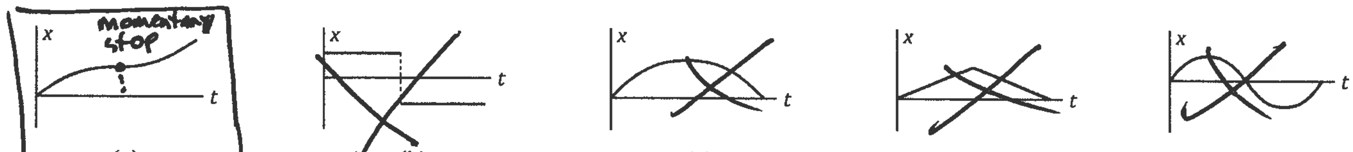
expression has correct units for describing speed V_0

Question value 8 points

(1) At right is the graph of velocity versus time for a particle mixing along the *x*-axis. The particle srats at the origin. Which of the graphs below *best* depicts the position of the particle, as a function of time?



Note v is always positive: object is always displacing toward larger x



(a) (b) (c) (d) (e)

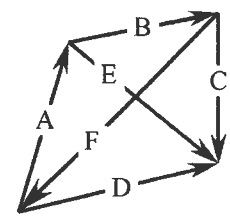
- does not start at origin
- "teleports" from pos to neg!

(c)–(e): particle is not always moving toward greater x

(a): ① Always moving toward larger, positive x
② starts fast, slows down to momentary stop, then starts speeding up again

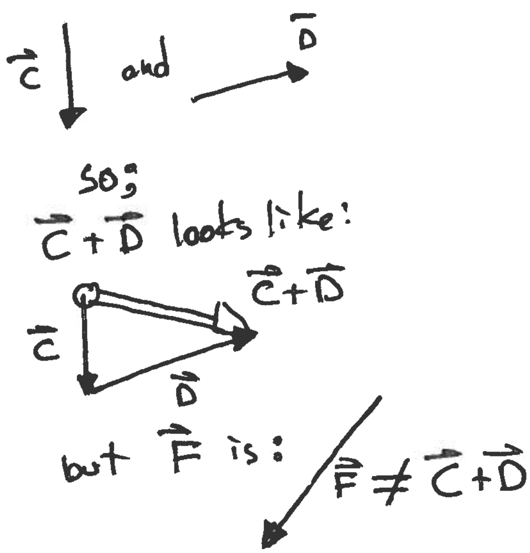
Question value 8 points

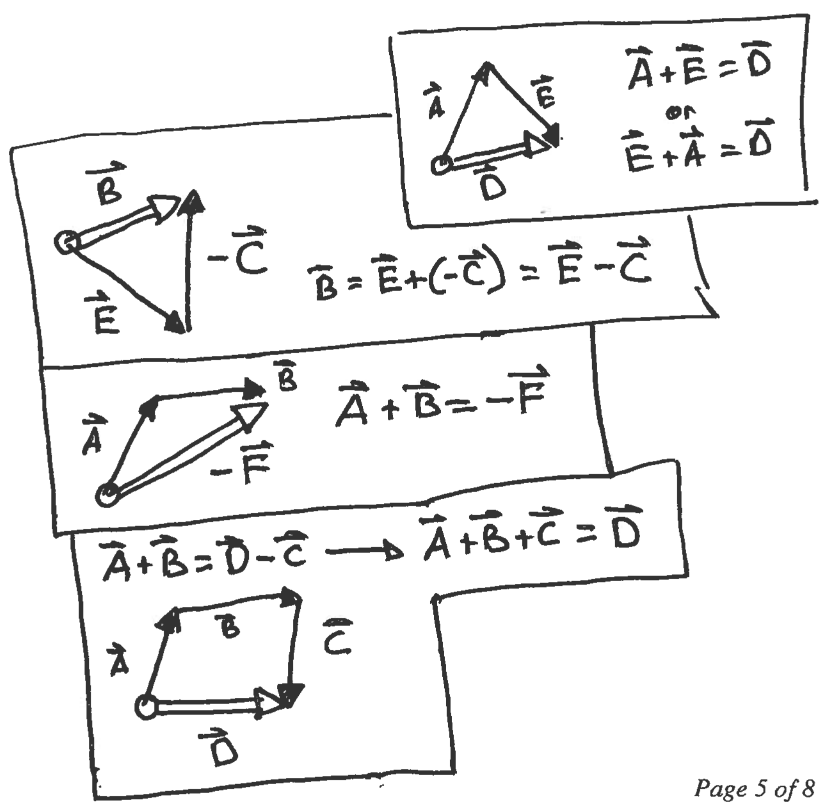
(2) Consider vectors $\vec{A}$ through $\vec{F}$, in the figure at right. Which of the expressions below is **NOT** a valid statement of vector arithmetic for the vectors involved?



sum vectors "tip-to-tail"

(a) $\vec{D} = \vec{E} + \vec{A}$ ✓
(b) $\vec{B} = \vec{E} - \vec{C}$ ✓
(c) $\vec{C} + \vec{D} = \vec{F}$ — not valid
(d) $\vec{A} + \vec{B} = -\vec{F}$ ✓
(e) $\vec{A} + \vec{B} = \vec{D} - \vec{C}$ ✓



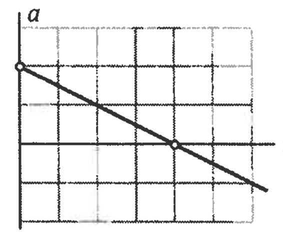


The next two questions involve the following situation:

Starting from rest at the origin, a particle moving along the x-axis experiences a non-uniform acceleration, given by the expression:

$$\vec{a}_x(t) = \langle +B \rangle + \langle -C \rangle\, t \qquad \text{for all } t \geq 0$$

where B and C are inherently positive constants.



[*Advice: before you start working the problems below, figure out what kind of units are possessed by B, and what kind of units are possessed by C.*]

Note: [units of B] = [accel]
[units of C]·[units of time] = [accel]
→ [units of B] = [units of C] • [units of time]
so [units of time] = [units of B] / [units of C]

⟹ Dimensional analysis says: for problems below, answers for Δt must be $\sim \frac{B}{C}$

Question value 4 points

(3) During what time interval will the particle be travelling in the positive direction?

(a) ~~$0 < t < C/2B$~~ wrong units

(b) $t > 2B/C$

(c) $0 < t < 2B/C$ [boxed]

(d) $0 < t < B/C$

(e) ~~$0 < t < \sqrt{C/B}$~~ wrong units

Recall: $\vec{\Delta v} = \int \vec{a}(t)\,dt$ (non constant accel!)

$\vec{v}(t) = \vec{v}_i + \int [B - Ct]\,dt = Bt - C\frac{t^2}{2}$ ($\vec{v}_i$ = zero)

so $v(t) = Bt - \frac{1}{2}t^2 = t\left(B - \frac{1}{2}Ct\right)$

Now, require v = positive: $t(B - \frac{1}{2}Ct) > 0 \rightarrow (B - \frac{1}{2}Ct) > 0$

$B > \frac{1}{2}Ct$

$\boxed{t < \frac{2B}{C}}$

Question value 4 points

(4) At what time (other than $t = 0$) will you find the particle at the origin?

(a) $t = 3B/C$ [boxed]

(b) $t = B/C$

(c) ~~$t = \sqrt{C/B}$~~ wrong units

(d) $t = 4B/C$

(e) ~~$t = 2C/B$~~ wrong units

Recall: $\vec{\Delta x} = \int \vec{v}(t)\,dt \rightarrow \vec{x}(t) - \vec{x}_i = \int \left[Bt - \frac{1}{2}Ct^2\right] dt$ ($\vec{x}_i$ = zero)

so $\vec{x}(t) = B\frac{t^2}{2} - \frac{1}{2}C\frac{t^3}{3}$

$\vec{x}(t) = t^2\left[\frac{1}{2}B - \frac{1}{6}Ct\right]$

so if $x = 0$, then either

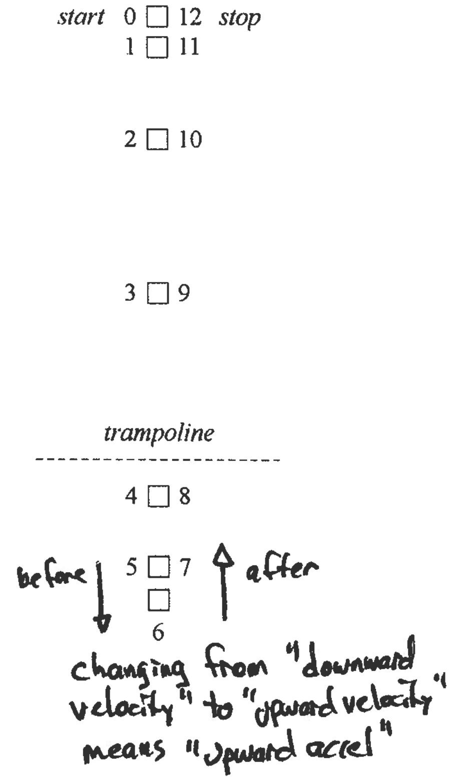
$t = 0$ [exclude that]

$\frac{1}{2}B - \frac{1}{6}Ct = 0$

$\frac{1}{2}B = \frac{1}{6}Ct \rightarrow \boxed{t = \frac{6B}{2C} = \frac{3B}{C}}$

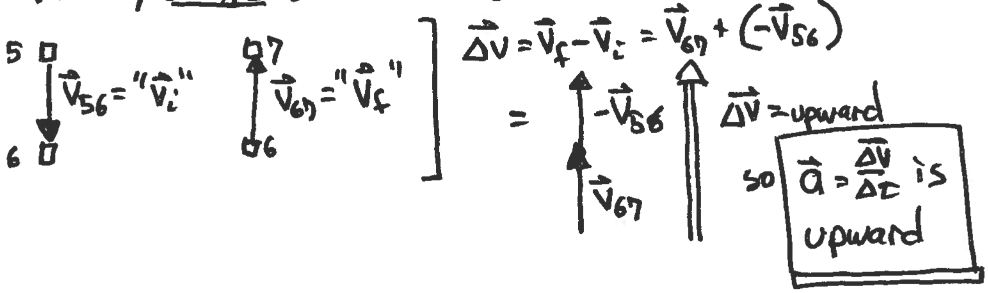
Question value 8 points

(5) An acrobat steps off a platform and drops straight down onto a trampoline. She stretches the trampoline down, comes to a momentary stop, and then bounces back up to her original height. At right is a motion diagram for the gymnast, with frames labelled in chronological order. Note that some frames are used twice; once on the way down (frames 0–5), and then again on the way up (frames 7-12). Which of the statements below best characterizes the acrobat's acceleration in frame 6?



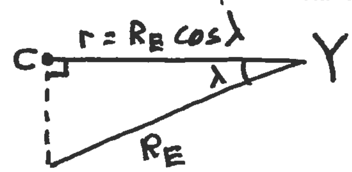
(a) Her acceleration cannot be obtained from the information provided.
(b) Her acceleration is non-zero and upward.
(c) Her acceleration is nonzero and downward.
(d) Her acceleration is zero.

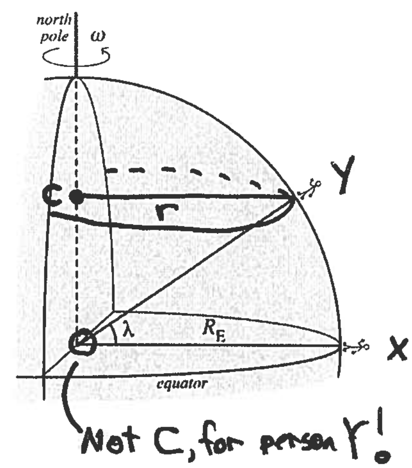
To learn about $\vec{a}$ "at frame 6", consider velocity changes over frames 5→6→7



Question value 8 points

(6) The Earth rotates with a period T, and thus all persons on Earth are experiencing uniform circular motion with angular speed $\omega = 2\pi/T$. Atlanta is located at latitude λ = 33.8° N. How does the radial (or "*centripetal*") acceleration of a person in Atlanta compare to the radial acceleration of a person standing on the equator?





(a) $a_{c,Atl} = 1.80\, a_{c,Eq}$

(b) $a_{c,Atl} = 0.691\, a_{c,Eq}$

(c) $a_{c,Atl} = 0.556\, a_{c,Eq}$

(d) $a_{c,Atl} = 1.20\, a_{c,Eq}$

(e) $a_{c,Atl} = 0.831\, a_{c,Eq}$

Note that center of circular path around Earth, C is not the center of the Earth

centripetal accel is $a_c = \frac{V_s^2}{r}$ where V_s = tangential speed $= \omega r$ and r = radius of circular path

so $a_c = \frac{(\omega r)^2}{r} = \omega^2 r$

→ For person X at equator, $a_{c,Eq} = \omega^2 R_E$

→ For person Y in Atlanta, $a_{c,Atl} = \omega^2 r = \omega^2 R_E \cos\lambda = (a_{c,Eq})\cos\lambda$

$$a_{c,Atl} = (\cos\lambda)\, a_{c,Eq} = 0.831\, a_{c,Eq}$$