Physics 2211M Test form **351** Name Solutions

Summer 2016

Test 3 Recitation Section (see back of test):__________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

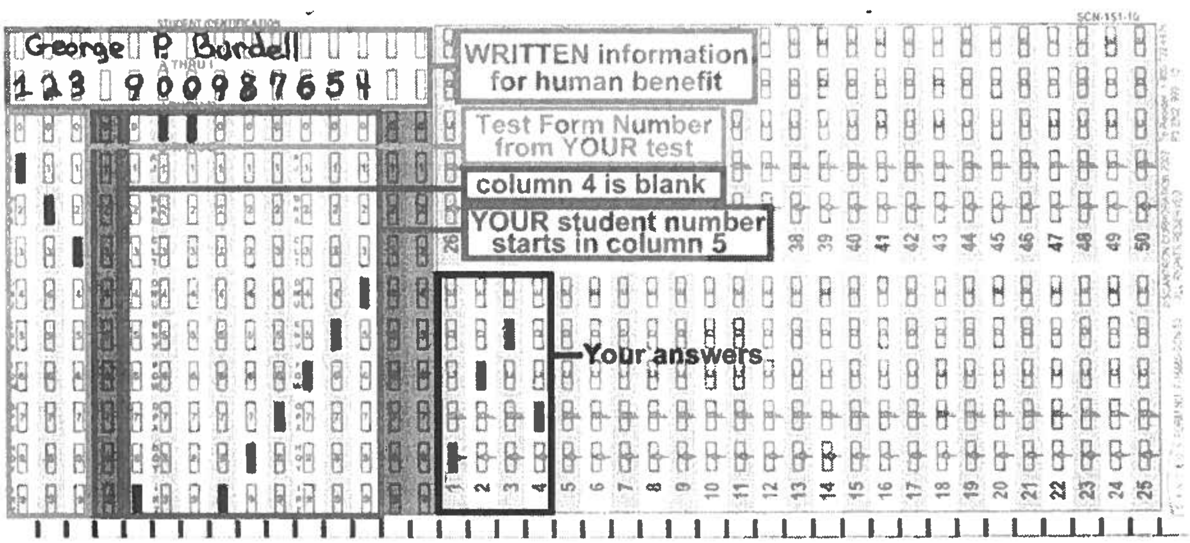
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

Your test form is: 351

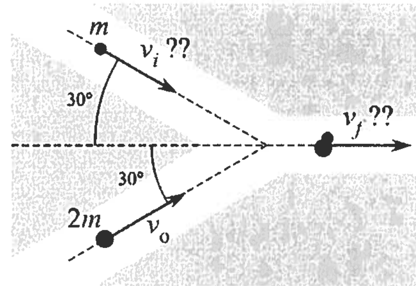


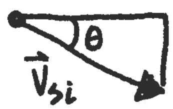
Our Final Exam will be held on:
Monday, August 01 at 8:00am

The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[I] *(20 points)* A sporstcar of mass m is travelling south of east with an undetermined speed, when it collides in a Y-intersection with a truck of mass $2m$ that is travelling north of east with a known speed v_o. As a result of the collision, the two vehicles stick together and skid out of the intersection travelling dues east.

Determine the initial speed of the sporstscar *before* the collision and the final speed of both vehicles immediately *after* the collision. Express each answer as a multiple of v_o.





$$\vec{V}_{si} = \langle +V_i\cos\theta\rangle\hat{i} + \langle -V_i\sin\theta\rangle\hat{j} = \langle +\tfrac{\sqrt{3}}{2}V_i\rangle\hat{i} + \langle -\tfrac{1}{2}V_i\rangle\hat{j}$$

$$\vec{V}_{Ti} = \langle +V_o\cos\theta\rangle\hat{i} + \langle +V_o\sin\theta\rangle\hat{j} = \langle +\tfrac{\sqrt{3}}{2}V_o\rangle\hat{i} + \langle +\tfrac{1}{2}V_o\rangle\hat{j}$$

Final state: <u>both</u> vehicles have $\vec{V}_f = \langle +V_f\rangle\hat{i} + \langle 0\rangle\hat{j}$

Momentum is conserved, separately, for each component direction

① Y-direction: $\underbrace{\langle +(2m)\tfrac{V_o}{2}\rangle}_{\text{truck}} + \underbrace{\langle -(m)\tfrac{V_i}{2}\rangle}_{\text{Sportscar}} = \langle (3m)\cdot 0\rangle$

$$\cancel{m}V_o - \tfrac{1}{2}\cancel{m}V_i = 0 \qquad V_o = \tfrac{1}{2}V_i \qquad \boxed{V_i = 2V_o}$$

initial speed of sportscar

② x-direction: $\langle +(2m)\tfrac{\sqrt{3}}{2}V_o\rangle + \langle +(m)\tfrac{\sqrt{3}}{2}V_i\rangle = \langle +(3m)V_f\rangle$

$$\sqrt{3}\,\cancel{m}V_o + \tfrac{\sqrt{3}}{2}\cancel{m}V_i = 3\cancel{m}V_f$$

$$\sqrt{3}V_o + \tfrac{\sqrt{3}}{2}(2V_o) = 3V_f$$

$$2\sqrt{3}\,V_o = 3V_f$$

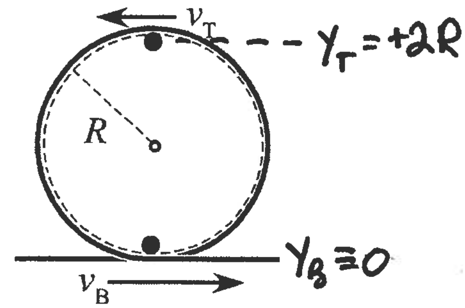
$$\boxed{V_f = \tfrac{2}{\sqrt{3}}V_o}$$

The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

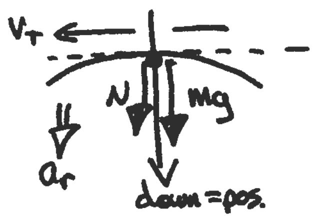
[II] *(20 points)* A block of mass m slides along a frictionless track through a circular loop-the-loop of radius R. It passes through the top of the loop with a speed v_T that is just *barely* enough to keep it in contact with the track.

Determine the speed v_B of the block at the bottom of the loop, and determine the apparent weight of the block at that moment. Express each answer in terms of m, g, and/or R, as appropriate.

Hint: at the bottom of the loop, you should treat the block as if it is still in the loop—you are calculating the situation just before it exits onto level track.



① At top: Circular motion dynamics



$$\Sigma F_r = m\vec{a}_r \rightarrow \langle +N\rangle + \langle +mg\rangle = m\langle +v_T^2/R\rangle$$

but at minimum safe speed, $N \rightarrow 0$ (barely "in contact")

$$\text{so } \cancel{m}g = \cancel{m}v_T^2/R \qquad \boxed{v_T^2 = gR}$$

dimensions: $\left[\frac{\text{length}}{\text{time}}\right]^2 \stackrel{?}{=} \left[\frac{\text{length}}{\text{time}^2}\right]\cdot\left[\text{length}\right]$

→ yes – dimensions match

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② Conservation of energy, Top to Bottom

$$K_T + U_T = K_B + U_B \longrightarrow \tfrac{1}{2}mv_T^2 + mg(2R) = \tfrac{1}{2}mv_B^2 + 0$$

$$\tfrac{1}{2}m(gR) + 2mgR = \tfrac{1}{2}mv_B^2 \rightarrow \boxed{v_B^2 = 5gR}$$

same dimension analysis

$$\text{so } \boxed{v_B = \sqrt{5gR}}$$

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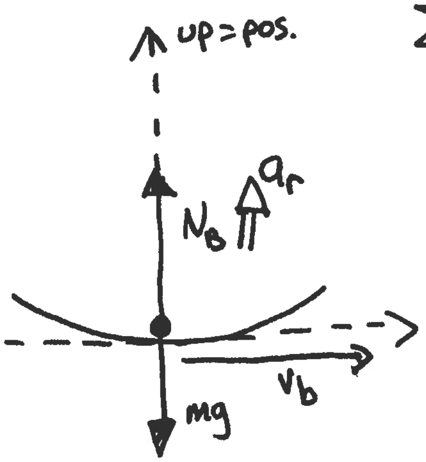
③ At bottom: circular motion dynamics (again)



$$\Sigma \vec{F}_r = m\vec{a}_r$$

$$\langle +N_B\rangle + \langle -mg\rangle = m\langle +a_r\rangle$$

[Note: N, a_r are NOT the same as at the top!]

$$N_B = mg + m\left(v_B^2/R\right) = mg + m\left(5gR/R\right)$$

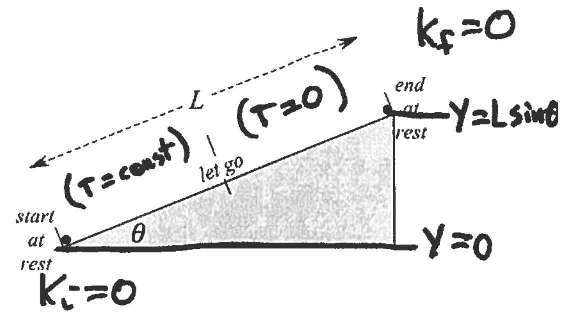
$$N_B = mg + 5mg = 6mg$$

⇒ Normal force creates perception of "weight" that is greater than true gravitational force

so perceived weight is $\boxed{6mg}$ → obvious force units

The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[III] *(20 points)* Starting from rest, a skier of mass m is pulled up a snowy 23.5° slope of total length L by a tow-rope that maintains a constant tension T. The coefficient of friction between the skis and the slope is $\mu_k = 0.220$. As the skier ascends, she gains speed. When she is exactly halfway to the top, she lets go of the tow-rope, and coasts to a stop right as she reaches the top of the slope.



Use the Work-Energy Principle to determine the tension in the rope while it is pulling the skier upslope. Express your answer as a multiple of the skier's weight, mg.

- For full trip, bottom-to-top: $\Delta K = K_f - K_i = 0$ (start and end at rest), with $K_f \to 0$, $K_i \to 0$

  while $\Delta U = U_f - U_i = (mgL\sin\theta - 0) \rightsquigarrow$ gain in PE!

  $\Rightarrow$ Mechanical Energy has increased: $\boxed{\Delta E_{mech} = \Delta K + \Delta U = +mgL\sin\theta}$

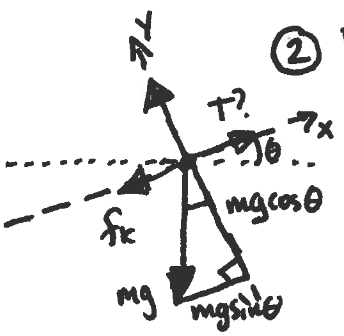
- How? ① Tension in pull-cord did positive work, adding energy.

  $$W_T = \vec{T}\cdot\vec{\Delta s} = \langle +T\rangle\cdot\langle +\tfrac{L}{2}\rangle = \boxed{+\tfrac{1}{2}TL}$$

  [Force units]·[length units]: correct units for work/energy

  work by constant force in 1 D motion

  $\tfrac{L}{2}$: only halfway!



  ② Friction did negative work, along entire route

  $$W_f = \vec{f}_k\cdot\vec{\Delta s} = \langle -\mu_k N\rangle\cdot\langle +L\rangle \quad \text{[again, constant force + 1D]}$$

  note that with or without tension along slope, normal force is given by $N = mg\cos\theta$, so $f_k = \mu_k mg\cos\theta$

  $$\boxed{W_f = -\mu_k mgL\cos\theta}$$

- Put it together: non-conservative work caused energy change

  $$W_T + W_f = \Delta E_{mech}$$

  $$+\tfrac{1}{2}T\cancel{L} - \mu_k mg\cancel{L}\cos\theta = +mg\cancel{L}\sin\theta \quad \text{length of slope drops out}$$

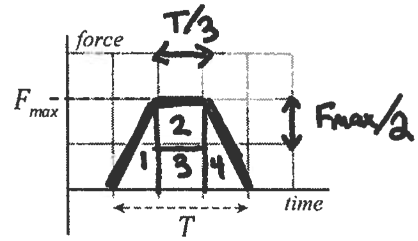
  $$\tfrac{1}{2}T = mg\sin\theta + \mu_k mg\cos\theta = mg(\sin\theta + \mu_k\cos\theta)$$

  so $$\boxed{\begin{aligned} T &= 2(\sin\theta + \mu_k\cos\theta)mg \\ &= 1.20\,mg \end{aligned}}$$

  T = multiple of grav force: obviously has correct dimension

Question value 8 points

(1) A rubber ball is dropped from height H onto the ground, and bounces back up in the air to a (lesser) height h. The graph at right depicts the normal force by the ground on the ball, while it is in contact. What constant normal force, acting for the same length of time T, would allow an identical ball, dropped from the same height H, to also rebound to height h?



(a) $\boxed{\frac{2}{3}F_{max}}$

(b) $\frac{3}{4}F_{max}$

(c) F_{max}

(d) $\frac{1}{2}F_{max}$

(e) $\frac{3}{2}F_{max}$

Velocity change at impact:
→ momentum change → impulse by ground on ball

$$\Delta\vec{p} = \vec{J} = \int \vec{F}\,dt = \text{area under force curve}$$

→ from graph, $|\vec{J}|$ = "4 squares"
→ height of one square: $\frac{F_{max}}{2}$
→ width " " " : $T/3$

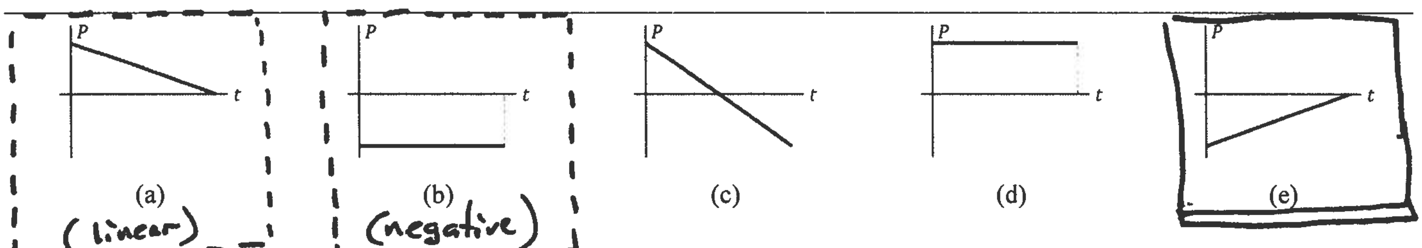
$$\text{so } J = 4\cdot\left(\frac{F_{max}}{2}\right)\left(\frac{T}{3}\right) = \frac{4}{6}F_{max}T = \frac{2}{3}F_{max}T$$

So: what constant force produces same impulse, in same time T?

$$F_{const}\cdot T = \frac{2}{3}F_{max}T \rightarrow \boxed{F_{const} = \frac{2}{3}F_{max}}$$

Question value 8 points

(2) A runaway freight train is travelling along a straight, frictionless stretch of track. Superman flies in front of the train, and pushes backward with a constant force F, bringing the train to a stop. Which of the graphs below best depicts the power delivery by Superman to the train, as a function of time?



① $P = \vec{F}\cdot\vec{v}$ → since $\vec{F}$ is opposite to $\vec{v}$, P = negative

② $|\vec{F}|$ = constant, but $|\vec{v}|$ decreases linearly to zero

⇒ $\boxed{\text{P(t) is a linear function that starts negative and decreases in magnitude to zero}}$

Each of the next four questions is worth 3 points.

In each of the following situations, two **identical** blocks are observed to collide while travelling on a frictionless surface. In each case, the initial state (top) and final state (bottom) are shown. For each situation, determine whether the collisions is:

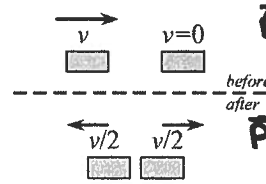
(a) A valid, perfectly inelastic collision.
(b) A valid, perfectly elastic collision.
(c) A valid collision that is partially elastic and partially inelastic—that is, something "in between" (a) and (b).
(d) An invalid collision—that is, a result that is actually impossible according to the laws of physics.

Hint: do not assume that each of the four descriptions above are applicable to exactly one of the situations below. Some descriptions above may be used more than once, and some may not be used at all.

Invalid? $\boxed{\text{is } \vec{P}_i \neq \vec{P}_f\text{?}}$ pick (d) Elastic? $\boxed{\text{is } K_i = K_f\text{?}}$ pick (b) Inelastic? $\boxed{\text{is } \vec{V}_{Af} = \vec{V}_{Bf}\text{?}}$ pick (a) Otherwise, pick (c)

(3)

(a) This is a perfectly inelastic collision.
(b) This is a perfectly elastic collision.
(c) This is a partially elastic / partially inelastic collision.
(d) This is an invalid collision.



$\vec{P}_i = \langle +mv \rangle$

$\vec{P}_f = \langle +mv/2 \rangle + \langle -mv/2 \rangle = 0$

$\boxed{\vec{P}_i \neq \vec{P}_f\,!}$

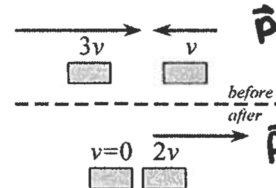
- $\vec{P}$ is not conserved: invalid collision

(4)

(a) This is a perfectly inelastic collision.
(b) This is a perfectly elastic collision.
(c) This is a partially elastic / partially inelastic collision.
(d) This is an invalid collision.

$K_i = \frac{1}{2}m(3v)^2 + \frac{1}{2}m(v)^2 = 4mv^2$

$K_f = 0 + \frac{1}{2}m(2v)^2 = 2mv^2$ $\boxed{K_i \neq K_f}$



$\vec{P}_i = \langle +3mv \rangle + \langle -mv \rangle = \langle +2mv \rangle$

$\vec{P}_f = 0 + \langle +2mv \rangle$

$\boxed{\vec{P}_i = \vec{P}_f}$

- $\vec{P}$ is conserved
- K is not conserved
- $\vec{V}_{Af} \neq \vec{V}_{Bf}$

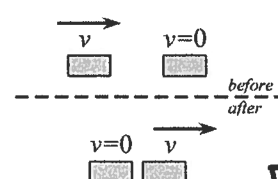
(5)

(a) This is a perfectly inelastic collision.
(b) This is a perfectly elastic collision.
(c) This is a partially elastic / partially inelastic collision.
(d) This is an invalid collision.

$K_i = \frac{1}{2}mv^2$

$K_f = \frac{1}{2}mv^2$

$\boxed{K_i = K_f}$



$\vec{P}_i = \langle +mv \rangle$

$\vec{P}_f = \langle +mv \rangle$

$\boxed{\vec{P}_i = \vec{P}_f}$

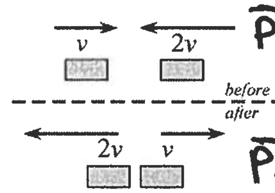
- $\vec{P}$ is conserved
- K is conserved

(6)

(a) This is a perfectly inelastic collision.
(b) This is a perfectly elastic collision.
(c) This is a partially elastic / partially inelastic collision.
(d) This is an invalid collision.

$K_i = \frac{1}{2}mv^2 + \frac{1}{2}m(2v)^2 = \frac{5}{2}mv^2$

$K_f = \frac{1}{2}m(2v)^2 + \frac{1}{2}mv^2 = \frac{5}{2}mv^2$ $\boxed{K_i = K_f}$



$\vec{P}_i = \langle +mv \rangle + \langle -2mv \rangle = \langle -mv \rangle$

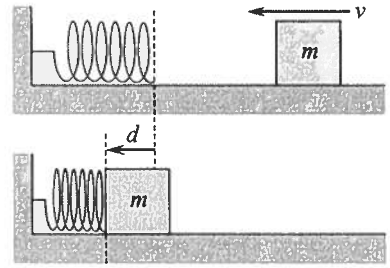
$\vec{P}_f = \langle -2mv \rangle + \langle +mv \rangle = \langle -mv \rangle$

$\boxed{\vec{P}_i = \vec{P}_f}$

- $\vec{P}$ is conserved
- K is conserved

Question value 8 points

(7) A block of mass m is initially moving with speed v along a frictionless surface, when it encounters a fixed spring. In the collision, the spring compresses by a distance d as it brings the block to a momentary stop—but immediately afterwards, the block will rebound in the opposite direction. What will be the magnitude of the block's acceleration at the moment it is stopped, with the spring at maximum compression?



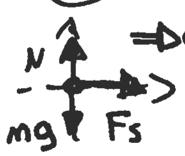
(a) ~~$a = 2mv/d^2$~~ wrong units for accel
(b) $a = 0$
$\boxed{\text{(c)}\ \ a = v^2/d}$
(d) $a = v^2/2d$
(e) ~~$a = mv/d^2$~~ wrong units

① Compression involves conservation of energy (frictionless surface!)

$$K_i + U_{si} = K_f + U_{sf} \rightarrow \frac{1}{2}mv^2 + 0 = 0 + \frac{1}{2}kd^2$$

$$\rightarrow \boxed{kd^2 = mv^2}$$

② 2nd law at moment of max compression

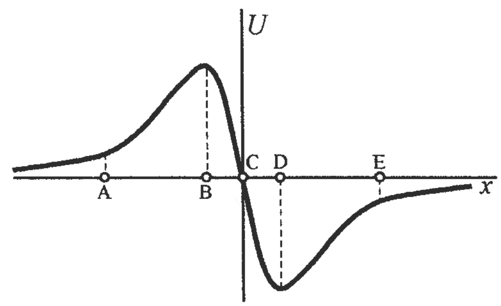


$$\Rightarrow a \quad \langle +F_s \rangle = m\langle +a \rangle \rightarrow +kd = ma \rightarrow kd \cdot d = mad$$

$$\cancel{m}v^2 = \cancel{m}ad$$

$$\boxed{a = v^2/d}$$

A particle moves along the x-axis, subject to a single force that is conservative. The potential energy function for this force is graphed at right. The following two questions refer to this graph.



Question value 4 points

(8) At which of the indicated locations is the particle in a position of *stable equilibrium*?

↳ U = min at position D

(a) Location C
$\boxed{\text{(b) Location D}}$
(c) Location A
(d) Location E
(e) Location B

$$\vec{F}(x) = \left\langle -\frac{dU}{dx} \right\rangle$$

• equilibrium: $\vec{F} = 0$

$$-\frac{dU}{dx} = 0 \Rightarrow U = \text{min/max}$$

but only $\boxed{U = \text{min}}$ is stable

Question value 4 points

(9) At which of the indicated locations is the particle experiencing a *positively-directed* force?

$\boxed{\text{(a) Location C}}$
(b) Location A
(c) Location B
(d) Location E
(e) Location D

Slope of U is negative at position C

• Force is positive when slope of U is negative