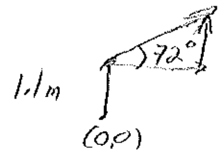
PHYS 2211 Traditional

Review Solutions

1) Home Plate is $X=0$ + $Y=0$



$X = X_0 + V_0 \cos\theta\, t + \frac{1}{2} a t^2$

$Y = Y_0 + V_0 \sin\theta\, t + \frac{1}{2} a t^2$

y-direction

a) $X = 41\cos 72^\circ\, t$

$X = 41\cos 72\,[5.07]$

$\boxed{X = 64.2\text{ m}}$

$0 = 1.1 + (41\sin 72)t - \frac{1}{2}(9.8)t^2$

$= 1.1 + 39t - 4.9t^2$

$t = \dfrac{-10.4 \pm \sqrt{(39)^2 - 4(-4.9)1.1}}{2(-4.9)}$

$t = -2.95 \qquad \boxed{t = 5.07}$

b) $V^2 = V_0^2 + 2a(y - y_0)$

$0 = (41\sin 72)^2 + 2(-9.8)(y - 1.1)$

$0 = 1520 - 19.6y + 21.56$

$\boxed{Y = 78.7\text{ m}}$

c) $X = X_0 + V_0\cos\theta\, t + \frac{1}{2} a t^2$

$X = [41\cos 72]\,t$

$X = [41\cos 72][3.8]$

$\boxed{X = 48\text{ m}}$

$y = y_0 + V_0 \sin\theta\, t + \frac{1}{2} a t^2$

$1.1 = 1.1 + [41\sin 72]t - 4.9t^2$

$[4.9t - 41\sin 72]\,t = 0$

$\boxed{t = \dfrac{41\sin 72}{4.9} = 3.80 \text{ sec}}$

2. $\frac{d^2x}{dt^2} = F/m = 1/5(4t^2)$ $\qquad$ $\frac{d^2y}{dt^2} = F/m = -1/5(3t)$

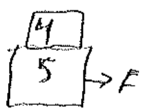
$\frac{dx}{dt} = v(t) = \frac{1}{5}\left(\frac{4}{3}t^3\right) + v_0$ $\qquad$ $\frac{dy}{dt} = -\frac{1}{5}\left(\frac{3}{2}t^2\right) + v_0$ $\qquad$ $v_0 = 0$

$\frac{dx}{dt} = \frac{1}{5}\left(\frac{4}{3}t^3\right)$ $\qquad$ $\frac{dy}{dt} = -\frac{1}{5}\left(\frac{3}{2}t^2\right)$

$x = \frac{1}{5}\left(\frac{1}{3}t^4\right) + x_0$ $\qquad$ $y = -\frac{1}{5}\left(\frac{1}{2}t^3\right) + x_0$ $\qquad$ $x_0 = 0$

$x = \frac{1}{15}t^4$ $\qquad$ $y = \frac{-1}{10}t^3$

$\boxed{\vec{v} = \left(\frac{4}{15}t^3\hat{i} - \frac{3}{10}t^2\hat{j}\right) \qquad x = \left(\frac{1}{15}t^4\hat{i} - \frac{1}{10}t^3\hat{i}\right)}$

3. 

$F_f = \mu_s mg$

$\frac{4}{9}(27) = \mu_s(4)(9.8)$

$\mu_s = .31$

$F = ma$

$27 = (4+5)a$

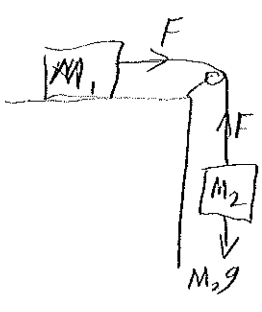
$a = \frac{27}{9}$

Force on small box by Friction

$F = (4)(27/9) = ma$

$(.31)(4)(9.8) = F_{max}$

$\boxed{12.2\ N}$

4. 

$F = M_1 a = M_2 g$

$-M_2 a = F - M_2 g$

$F = M_1 a$

$-M_2 a = m_1 a - M_2 g$

$(M_1 + M_2)a = M_2 g$

$a = \left(\frac{M_2}{M_1 + M_2}\right) g$

$\boxed{x = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{M_2}{M_1 + M_2}\right) g}$

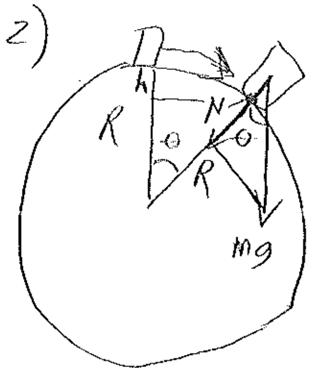
Rotational Motion

1.) $\omega = \omega_0 + \alpha t$ $\qquad$ $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega = (5)(15)$ $\qquad$ $\theta = \frac{1}{2}(5)(15)^2$

$\omega = 75$ rads/sec $\qquad$ $\theta = 563$ rads

2)



$$\frac{MV^2}{R} - Mg\cos\theta = 0 \qquad V^2 = 2gh$$

$$= 2gR(1-\cos\theta)$$

$$\frac{2gR(1-\cos\theta)}{R} - g\cos\theta = 0$$

$$2 = 3\cos\theta$$

$$\cos\theta = \frac{2}{3}$$

$$\boxed{\theta = \cos^{-1}\frac{2}{3}}$$

3) $A \xrightarrow[2m]{}$ $\qquad$ $V_0 = 6\,m/s$

momentum is conserved

For a particle

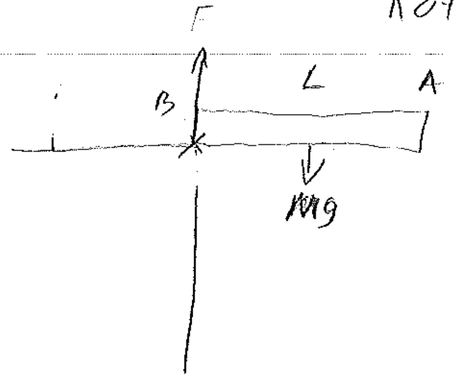
$L = I\omega = mRV$

$m_0 R_0 V_0 = m R_f V_f$

$(2)(6) = R(20)$

$\boxed{R = 0.6\ m}$

Rotational Motion

4) 

M = mass

$\tau = rF\sin\theta$

$\tau = \frac{L}{2} mg(1)$

$\boxed{\tau = \frac{Lmg}{2}}$

$\tau = I\alpha$

$\alpha = \frac{\tau}{I} = \frac{\frac{1}{2}LMg}{\frac{ML^2}{3}}$

$= \frac{LMg}{2} \cdot \frac{3}{ML^2}$

$\boxed{\alpha = \frac{3g}{2L}}$

$a = r\alpha$

$a = \left(\frac{L}{2}\right)\left(\frac{3g}{2L}\right)$

$\boxed{a = \frac{3g}{4}}$

$F - mg = \frac{-3mg}{4}$

$F = mg\left(1 - \frac{3}{4}\right)$

$\boxed{F = \frac{1}{4}mg}$

Momentum and Impulse

1) $M \longleftarrow \bullet \longrightarrow M$

$$M(50) - M(100) + 2MV = 0$$

$$2V = 50$$

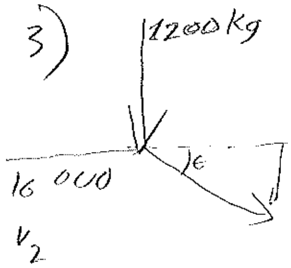
$$V = 25\,m/sec$$

in the direction of the slower piece

2) $$F = \frac{dP}{dt} = \frac{V\,dm}{dt}$$

$$= \frac{V\,\pi\left(\frac{D}{2}\right)^2 dL}{dt}$$

$$\boxed{F = \frac{\pi D^2 V^2}{4}}$$

3) 

$$P^2 = (1200V_1)^2 + (16000V_2^2)$$

$$P = \sqrt{(1200V_1)^2 + (16000V_2)^2}$$

$$\tan\theta = \frac{1200V_1}{16000V_2}$$

$$\boxed{\tan\theta = \frac{3}{40}\frac{V_1}{V_2}}$$

4a) $\left(\frac{Nmu}{M} = V\right)$ flat car

4c) All men jump at once yeilds a higher velocity

4b) $$mu = [M + (N-1)m]V$$

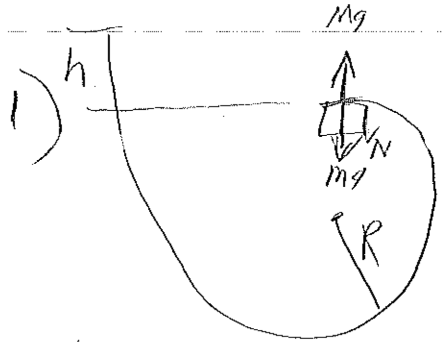
$$mu = [M + (N-2)m]V + [M + (N-1)m]V$$

$$Nmu = NMV + m\left(\sum_{i=0}^{N-1} N - i\right)V$$

$$V = \frac{Nmu}{NM + m\sum_{i=1}^{N}(i-1)}$$

Work + Energy

1)



$$-Mg + N = Mg$$

$$N = 2Mg$$

$$\frac{M(2gh)}{R} = 2Mg$$

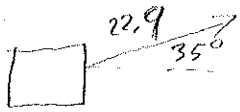
$$N = \frac{mv^2}{R}$$

$$v^2 = 2gh$$

$$h = R$$

Drop from a height of R above the top of to loop or 3R from the bottom.

2)



$$w = \vec{F} \cdot \vec{d}$$

$$w = (22.9)(129)\cos 35^\circ$$

$$= 2.69 \text{ KJ}$$

4) $E = mgh = \text{work}$

$$V_f = \sqrt{2gh}$$

$$\bar{V} = \frac{V_0 + V_f}{2}$$

$$\bar{V} = \frac{0 + \sqrt{2gh}}{2}$$

$$Y = \bar{V} t$$

$$t = \frac{Y}{\bar{V}} = \frac{[0.6]2}{\sqrt{2gh}}$$

$$\bar{P} = \frac{mgh}{t}$$

$$\bar{P} = \frac{mgh}{\frac{(0.6)2}{\sqrt{2gh}}}$$

$$= \frac{(55)[(9.8)(0.8)]^{3/2}(\sqrt{2})}{(0.6)2}$$

$$\boxed{\bar{P} = 1.42 \text{ K Watts}}$$

3. A projectile with mass m is fired from the surface of the Earth at an angle of α to the vertical. Its initial speed is $v_0 = \sqrt{GM_e/R_e}$. How high does the projectile rise? Neglect air resistance and the rotation of the Earth.

Because this object is launched at an angle from the Earth's surface, it can be treated as moving in a non-circular orbit that happens to intersect the Earth. In a system consisting of the object and the Earth, there are no external forces to do work or exert torques, so both energy and angular momentum are conserved. Look first at angular momentum, to find the object's speed at its greatest distance. In terms of magnitudes

$$L_i = L_f \qquad \Rightarrow \qquad r_i m v_i \sin\alpha = r_f m v_f \sin 90^\circ \qquad \Rightarrow \qquad v_f = v_i\left(\frac{r_i}{r_f}\right)\sin\alpha$$

Note that at its greatest distance, the object must be moving in a direction perpendicular to a line from the Earth's center. If the velocity had any component along that line, the object would be getting closer to or farther from the Earth, and so would not be at its greatest distance.

Now look at energy. Because the object may go a great distance from the Earth, do not assume g is constant. Use the Universal Gravitation potential energy. There are no thermal energy changes.

$$W_{\text{ext}} = \Delta K + \Delta U + \Delta E_{\text{th}} \qquad \Rightarrow \qquad 0 = \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right) + \left(\frac{-GMm}{r_f} - \frac{-GMm}{r_i}\right) + 0$$

Substitute the expression found above for the final speed. The mass of the object cancels. Multiplying by 2 makes things neater.

$$0 = \left(\tfrac{1}{2}m\left[v_i\left(\frac{r_i}{r_f}\right)\sin\alpha\right]^2 - \tfrac{1}{2}mv_i^2\right) + \left(\frac{-GMm}{r_f} - \frac{-GMm}{r_i}\right) \qquad \Rightarrow \qquad 0 = \left(v_i^2\frac{r_i^2}{r_f^2}\sin^2\alpha - v_i^2\right) - \frac{2GM}{r_f} + \frac{2GM}{r_i}$$

Substitute the given expression for the initial speed. The object is initially a distance R from the Earth's center.

$$0 = \left(\left[\sqrt{GM/R}\right]^2\frac{R^2}{r_f^2}\sin^2\alpha - \left[\sqrt{GM/R}\right]^2\right) - \frac{2GM}{r_f} + \frac{2GM}{R} = \left(\frac{GMR}{r_f^2}\sin^2\alpha - \frac{GM}{R}\right) - \frac{2GM}{r_f} + \frac{2GM}{R}$$

The Universal Gravitation constant cancels. The mass of the Earth cancels.

$$0 = \frac{R}{r_f^2}\sin^2\alpha - \frac{1}{R} - \frac{2}{r_f} + \frac{2}{R} \qquad \Rightarrow \qquad 0 = \frac{R}{r_f^2}\sin^2\alpha + \frac{1}{R} - \frac{2}{r_f}$$

Multiply by Rr_f^2.

$$0 = R^2\sin^2\alpha + r_f^2 - 2Rr_f \qquad \Rightarrow \qquad r_f^2 - 2Rr_f + R^2\sin^2\alpha = 0$$

which is quadratic in r_f. So

$$r_f = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \qquad \text{where} \qquad A = 1 \quad B = -2R \quad C = R^2\sin^2\alpha$$

Then

$$r_f = \frac{2R \pm \sqrt{(-2R)^2 - 4R^2\sin^2\alpha}}{2} = \frac{2R \pm \sqrt{4R^2 - 4R^2\sin^2\alpha}}{2} = \frac{2R \pm 2R\sqrt{1-\sin^2\alpha}}{2} = R \pm R\sqrt{1-\sin^2\alpha}$$

As the object starts at a distance R from the center of the Earth, the greatest distance must be more than that, so choose the positive solution.

$$r_f = R + R\sqrt{1-\sin^2\alpha}$$

But that's the greatest distance from the center of the Earth. The greatest height achieved above the surface must be R less than that, so

$$h = R\sqrt{1-\sin^2\alpha} = R\sqrt{\cos^2\alpha} = R\cos\alpha$$