Physics 2211 BCD
Spring 2015
Test 1

Test form **411**

Name Official Solutions

Recitation Section (see back of test):__________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

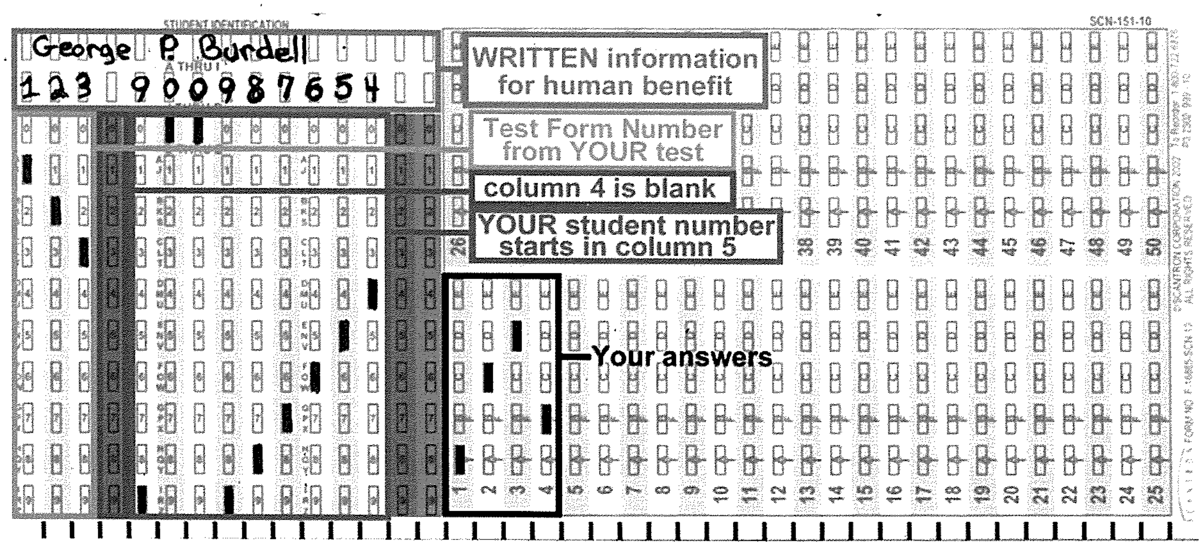
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

Your test form is: 411



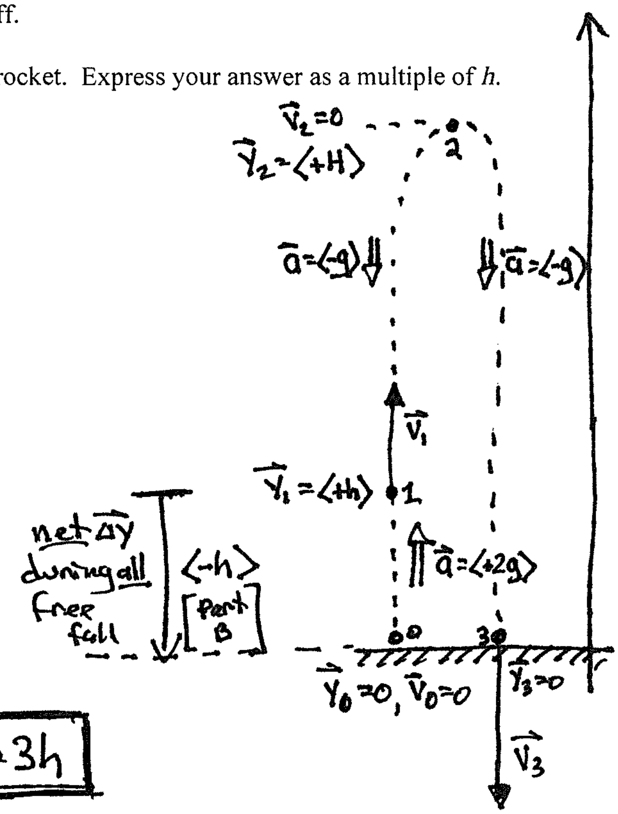
Our next test will be on Monday, February 16!

$\boxed{g = 9.80\ m/s^2}$

The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[I] A rocket is fired from rest, starting at ground level. While the rocket motor is on, the rocket experiences an upward acceleration of magnitude $a_{up} = 2g$. (This is the net acceleration, after the effects of gravity have been taken into account.) After reaching a height h above ground, the rocket motor cuts off.

(A) *(10 points)* Find an expression for the maximum height H reached by the rocket. Express your answer as a multiple of h.



① Powered ascent: $\vec{a} = \langle +2g \rangle$

$\vec{V} = \cancel{\vec{V}_{0y}} + \langle +2g \rangle \Delta t$ and $\vec{Y} = \cancel{\vec{Y}_{0y}} + \cancel{\vec{V}_0 \Delta t} + \frac{1}{2}\langle +2g \rangle \Delta t^2$

AND: "speed equation"

$V_f^2 = \cancel{V_{0y}^2} + 2\langle +2g \rangle \langle \Delta y \rangle$

→ using $V_f = V_1$ and $\Delta y = \langle +h \rangle$, we get: $V_1^2 = 2\langle +2g \rangle \langle +h \rangle$

$\boxed{V_1^2 = 4gh \text{ or } V_1 = 2\sqrt{gh}}$

② Non-powered ascent from 1 to 2: $\vec{a} = \langle -g \rangle$

Use speed equation again: $V_2 = 0$ at apex, so

$V_2^2 = 0 = V_1^2 + 2\langle -g \rangle \langle \Delta Y_{12} \rangle$

$\Delta Y_{12} = \dfrac{V_1^2}{2g} = \dfrac{4gh}{2g} = 2h$ or $\boxed{\vec{\Delta Y}_{12} = \langle +2h \rangle}$

so $\vec{Y}_2 = \vec{Y}_1 + \vec{\Delta Y}_{12} = \langle +h \rangle + \langle +2h \rangle = \langle +3h \rangle \Rightarrow \boxed{H = 3h}$

(B) *(10 points)* Suppose that the rocket's motor cut-off occurs at a height h = 1250 m. For what total time Δt_{TOT} will the rocket be airborne (i.e. including its powered ascent, its free-fall ascent, and its free-fall descent)?

[No penalty here if you correctly use a wrong answer from Part A (eg, wrong H)]

① Powered ascent: use position equation: $\vec{\Delta y} = \frac{1}{2}\langle +2g \rangle \Delta t^2 \rightarrow \langle +h \rangle = \frac{1}{2}\langle +2g \rangle \Delta t_{01}^2$

$\boxed{\Delta t_{01} = \sqrt{\frac{h}{g}}}$

② Free fall – includes both non-powered ascent, and descent to ground

$\vec{\Delta y} = \vec{V}_i \Delta t + \frac{1}{2}\vec{a}\Delta t^2$ → ($\vec{\Delta y} = \vec{Y}_3 - \vec{Y}_1 = \langle 0 \rangle - \langle +h \rangle = \langle -h \rangle$; $\vec{V}_i = \langle +V_1 \rangle = \langle +2\sqrt{gh} \rangle$ + $\vec{a} = \langle -g \rangle$)

$\langle -h \rangle = \langle +2\sqrt{gh} \rangle \Delta t_{13} + \frac{1}{2}\langle -g \rangle \Delta t_{13}^2$

$\boxed{\frac{1}{2}g\,\Delta t_{13}^2 - 2\sqrt{gh}\,\Delta t_{13} - h = 0}$

Quadratic Equation!

[I'm not afraid of it ... are you?]

$\Delta t_{13} = \dfrac{+2\sqrt{gh} \pm \sqrt{4gh - 4(\frac{1}{2}g)(-h)}}{2(\frac{1}{2}g)} = \dfrac{2\sqrt{gh} \pm \sqrt{6gh}}{g}$

↳ must be positive: choose ⊕ root

$\Delta t_{13} = (2+\sqrt{6})\sqrt{\frac{h}{g}}$

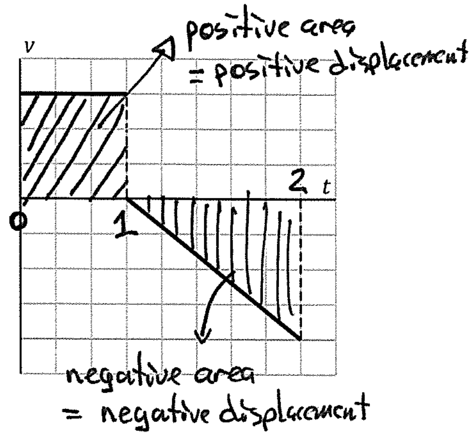
$\Delta t_{TOT} = \Delta t_{01} + \Delta t_{13} = \sqrt{\frac{h}{g}} + (2+\sqrt{6})\sqrt{\frac{h}{g}} = \boxed{(3+\sqrt{6})\sqrt{\frac{h}{g}}}$

$\boxed{\Delta t_{TOT} = 61.5 \text{ sec} = 1.03 \text{ min}}$

The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[II] The graph at right displays the velocity-vs-time plot for a particle moving in one dimension. Each vertical unit on the graph corresponds to 5.0 m/s of velocity, and each horizontal unit of the graph corresponds to 5.0 seconds of elapsed time.

(15 points) Determine the average velocity of the particle over the full 40-second interval displayed in the plot.



definition: $\vec{V}_{av} = \dfrac{\vec{\Delta x}}{\Delta t}$, in one dimension

$\vec{\Delta x}$ = displacement = area under velocity graph

- $\vec{\Delta x}_{01} = \underbrace{+}_{\text{pos. area}}(\text{height})\cdot(\text{width}) = +(3)(3)$

  $\vec{\Delta x}_{01} = +9$ squares

- $\vec{\Delta x}_{12} = \underbrace{-}_{\text{neg area}}\underbrace{\tfrac{1}{2}}_{\text{triangle}}(\text{height})\cdot(\text{width}) = -\tfrac{1}{2}(4)(5) = -10$ squares

$\Rightarrow$ Total area is $\vec{\Delta x}_{02} = +9 - 10 = -1$ square

Also: each square has "area": $(5\text{ m/s})\cdot(5\text{ sec}) = 25\text{m}$

so $\boxed{\vec{\Delta x}_{02} = \langle -25\text{ m}\rangle}$

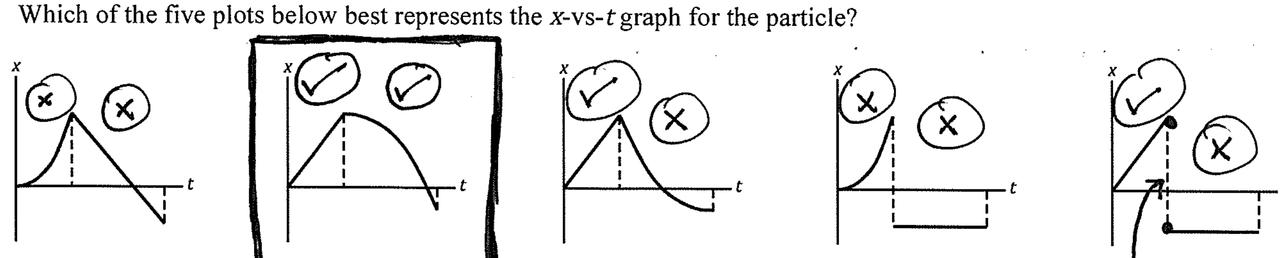
total elapsed time is $\Delta t_{02} = 8 \text{ squares} \times \left(\dfrac{5\text{sec}}{1\text{square}}\right) = 40$ seconds

hence $\vec{V}_{av} = \dfrac{\langle -25\text{m}\rangle}{40\text{ sec}} = \langle -0.625\text{ m/s}\rangle$

Since grid ~~size~~ has two digit accuracy, proper answer is $\boxed{\vec{V}_{av} = \langle -0.63\text{ m/s}\rangle}$

Question value 5 points — mark your answer in space #1 on the answer card.

(1) Which of the five plots below best represents the x-vs-t graph for the particle?



(a) (b) (c) (d) (e)

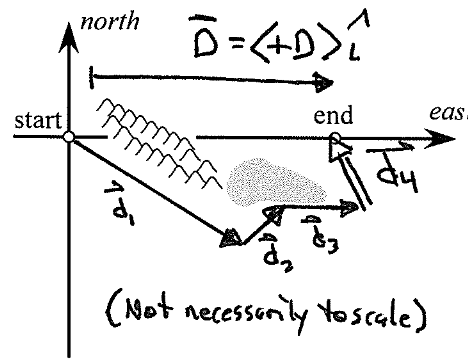
teleportation is unphysical!

① constant velocity during first interval: linearly increasing $\vec{x}(t)$

② increasingly negative velocity in second: downcurving, quadratic $\vec{x}(t)$

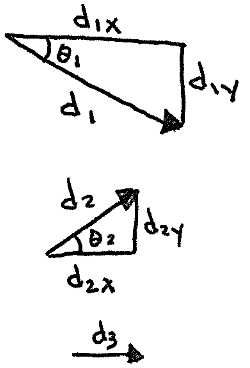
The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

[III] You are hiking cross-country, aiming for a wilderness campsite that is a distance D =12 km due east of your starting point. A difficult, rocky ridge blocks your direct path, so you start by traveling in a direction θ_1 = 23° south of east. After travelling a distance d_1 = 9.0 km, you turn and travel in a direction θ_2 = 33° north of east. After travelling d_2 = 4.0 km, you find your way blocked by an impassable swamp. You travel due east (θ_3 = 0.0°) along the edge of the swamp for d_3 = 2.0 km, at which point you reach a clearing and the swamp ends.



(15 points) What displacement will take you directly from the clearing at end of the swamp to the campsight?

$$\vec{d}_1 = (d_1 @ \theta_1) = \langle +d_1\cos\theta_1 \rangle \hat{\imath} + \langle -d_1 \sin\theta_1 \rangle \hat{\jmath} = \langle 8.285\text{ km} \rangle \hat{\imath} \langle -3.517\text{ km} \rangle \hat{\jmath}$$



$$\vec{d}_2 = \langle +d_2\cos\theta_2 \rangle \hat{\imath} + \langle +d_2\sin\theta_2 \rangle \hat{\jmath} = \langle +3.355\text{ km} \rangle \hat{\imath} + \langle +2.179\text{ km} \rangle \hat{\jmath}$$

$$\vec{d}_3 = \langle +d_3 \rangle \hat{\imath} + \langle 0 \rangle \hat{\jmath} = \langle +2.0\text{ km} \rangle \hat{\imath}$$

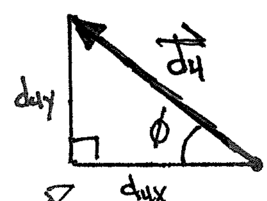
excessive precision to prevent subsequent rounding errors

Find displacement $\vec{d}_4$ = unknown

BUT: $\vec{d}_1 + \vec{d}_2 + \vec{d}_3 + \vec{d}_4 = \vec{D} = \langle +12\text{ km} \rangle \hat{\imath}$

$$\vec{d}_4 = \vec{D} - \vec{d}_1 - \vec{d}_2 - \vec{d}_3$$

$$= \left[\langle +12\text{ km} \rangle \hat{\imath} + \langle 0 \rangle \hat{\jmath}\right]$$
$$- \left[\langle +8.285\text{ km} \rangle \hat{\imath} + \langle -3.517\text{ km} \rangle \hat{\jmath}\right]$$
$$- \left[\langle +3.355\text{ km} \rangle \hat{\imath} + \langle +2.179\text{ km} \rangle \hat{\jmath}\right]$$
$$- \left[\langle +2.0\text{ km} \rangle \hat{\imath} + \langle 0 \rangle \hat{\jmath}\right]$$



$|\vec{d}_4| = \sqrt{d_{4x}^2 + d_{4y}^2}$

$\boxed{d_4 = 2.1\text{ km}}$

at angle $\phi = \tan^{-1}\left(\frac{d_{4y}}{d_{4x}}\right)$

$\boxed{\phi = 39^\circ \text{ N of W}}$

$$\vec{d}_4 = \langle -1.640\text{ km} \rangle \hat{\imath} + \langle +1.338\text{ km} \rangle \hat{\jmath} \Rightarrow \boxed{\vec{d}_4 = \langle -1.6\text{ km} \rangle \hat{\imath} + \langle +1.3\text{ km} \rangle \hat{\jmath}}$$

Cartesian form is an acceptable answer

Question value 5 points — mark your answer in space #2 on the answer card.

(2) Compare your average speed during the trip, v_{av}, to the magnitude of your average velocity during the trip, $|\vec{v}_{av}|$.

(a) It must be true that $v_{av} = |\vec{v}_{av}|$.
(b) It might be true that $v_{av} < |\vec{v}_{av}|$.
(c) It must be true that $v_{av} < |\vec{v}_{av}|$.
(d) It must be true that $v_{av} > |\vec{v}_{av}|$.
(e) It might be true that $v_{av} > |\vec{v}_{av}|$.

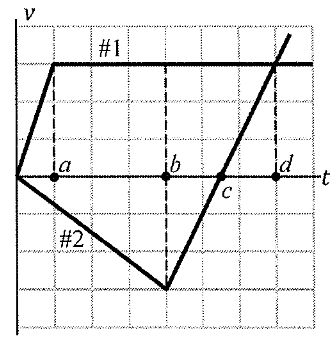
Not even counting d_4:

distance travelled on 1st 3 legs = 15 km > D

$$|\vec{V}_{av}| = \left|\frac{\Delta \vec{r}}{\Delta t}\right| = \frac{D}{\Delta t_{TOT}}$$

$$\text{but } V_{av} = \frac{d_1 + d_2 + d_3 + d_4}{\Delta t_{TOT}} > \frac{D}{\Delta t_{TOT}} = |\vec{V}_{av}|$$

$$\boxed{V_{av} > |\vec{V}_{av}|}$$



Two cars move along the x-axis, with the velocities plotted as functions of time in the graph at right. Car #1 starts at initial position $\vec{x}_{1i} = \langle 0 \rangle$, and Car #2 starts at initial position $\vec{x}_{2i} = \langle -D \rangle$.

Question value 5 points

(3) At which of the times indicated in the graph will Car #2 *first* begin to reduce its distance from Car #1?

ie when does #2 "start catching up"

(a) At time *a*.

(b) At none of the times shown in the plot.

(c) At time *b*.

(d) At time *c*.

(e) At time *d*.

Note well: Car#2 starts out "behind"

- from $t=0$ to $t=t_c$: #2 moves in negative direction, while #1 moves in positive direction ⇒ distance between cars is definitely increasing
- between t_c and t_d, both cars are moving in positive direction, but #1 is moving faster ⇒ separation distance is still increasing

⇒ only after t_d is #2 moving faster, in same direction as #1

Question value 5 points

(4) At which of the times indicated in the graph will Car #2 *overtake* Car #1?

(a) At none of the times shown in the plot.

(b) At time *d*.

(c) At time *a*.

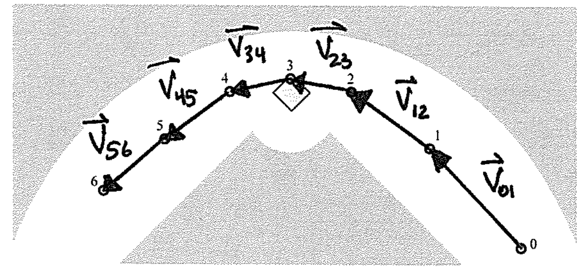
(d) At time *b*.

(e) At time *c*.

clearly impossible — #2 has been moving away from #1, traveling in the wrong direction!

⇒ at time t_d, #2 starts to catch up, but still has a Long way to go, to actually pass #1!

The motion diagram at right portrays a baseball player rounding second base, as seen from directly above. Please use this diagram to answer the four questions below.



Question value 5 points

(5) During what two intervals is the baserunner *gaining* speed?

(a) Between frames 4 and 6.
(b) Between frames 2 and 4.
(c) Between frames 3 and 5.
(d) Between frames 0 and 2.
(e) Between frames 1 and 3.

increasing speed means magnitude of succesive $\Delta\vec{r}$'s (or $\vec{V}$s) gets larger

$\rightarrow |\Delta\vec{r}_{45}| > |\Delta\vec{r}_{34}|$, so $|\vec{V}_{45}| > |\vec{V}_{34}|$

$\boxed{\text{in intervals } 3\rightarrow4,\ 4\rightarrow5, \text{ speed increases}}$

Question value 5 points

(6) During what two intervals does the baserunner experience negative acceleration?

(a) Between frames 0 and 2.
(b) Between frames 2 and 4.
(c) There is no valid answer to this question.
(d) Between frames 1 and 3.
(e) Between frames 3 and 5.

"Negative" only has meaning for a vector when it is compared to an established coordinate system

$\Rightarrow$ With no coord axes given, $\boxed{\text{"negative acceleration" has no meaning}}$

[Also: "negative acceleration" is **NOT** synonymous with "slowing down"]

Question value 5 points

(7) During what two intervals is the baserunner's velocity most nearly constant?

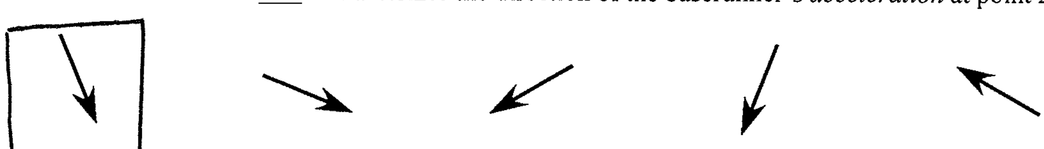
(a) Between frames 4 and 6.
(b) Between frames 2 and 4.
(c) Between frames 1 and 3.
(d) Between frames 0 and 2.
(e) Between frames 3 and 5.

In the diagram above it is straightforward to observe that $\vec{V}_{45}$ and $\vec{V}_{56}$ have nearly the same magnitude and direction

$$\boxed{\vec{V}_{45} \approx \vec{V}_{56}}$$

Question value 5 points

(8) Which of the arrows below best characterizes the direction of the baserunner's *acceleration* at point 2 ?



(a) (b) (c) (d) (e)

(a) Slowing down and turning left

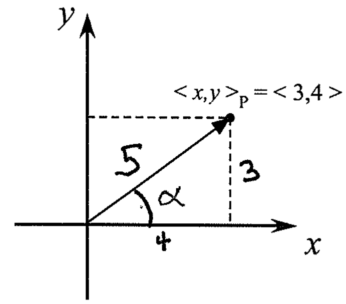
(b) This $\vec{a}$ describes "slowing down"

(d) This $\vec{a}$ describes "turning left"

$\Rightarrow$ Perform geometrical construction:

$\vec{a}_2 \propto \vec{V}_{32} - \vec{V}_{21} = \vec{V}_{32} + (-\vec{V}_{21})$

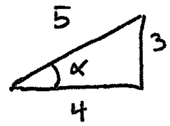
Point P is located at coordinates $\langle x, y\rangle = \langle +4.00\text{ cm}, +3.00\text{ cm}\rangle$ (top right). A second, rotated coordinate system is established—the uv-axes, which have been rotated by and angle $\theta = 16.2°$ clockwise from the xy-axes (bottom right).



It may not appear that the position vector $\vec{r}_P$ is the same in both figures, but it is!

Question value 5 points

(9) What is the u-coordinate of the same point P?



$\tan\alpha = \frac{3}{4}$

$\boxed{\alpha = 36.9°}$

(a) $\vec{r}_u = \langle +3.84\text{ cm}\rangle$

$\boxed{\text{(b) } \vec{r}_u = \langle +3.00\text{ cm}\rangle}$

~~(c) $\vec{r}_u = \langle +4.23\text{ cm}\rangle$~~ See Note*

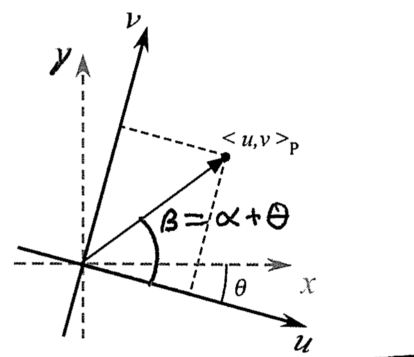
(d) $\vec{r}_u = \langle +3.72\text{ cm}\rangle$

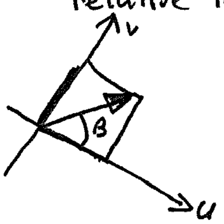
~~(e) $\vec{r}_u = \langle +4.80\text{ cm}\rangle$~~ See Note

1st, note: we have a 3-4-5 triangle!

For uv-axis rotated $\theta = 16.2°$ clockwise

$\Rightarrow$ Vector $\vec{r}_P$ makes angle $\beta = \alpha + \theta = 53.1°$ relative to u-axis





$\vec{r}_u = |\vec{r}|\cos 53.1°$

$= (5)\left(\frac{3}{5}\right) = \boxed{3.00\text{cm}}$

Question value 5 points

(10) What is the v-coordinate of the same point P?

$\boxed{\text{(a) } \vec{r}_v = \langle +4.00\text{ cm}\rangle}$

~~(b) $\vec{r}_v = \langle +2.88\text{ cm}\rangle$~~ See Note*

(c) $\vec{r}_v = \langle +3.12\text{ cm}\rangle$

(d) $\vec{r}_v = \langle +3.57\text{ cm}\rangle$

~~(e) $\vec{r}_v = \langle +2.40\text{ cm}\rangle$~~ See Note*

* Note well: v-axis has been rotated toward point P, as compared to y-axis

$\Rightarrow$ v-component must be larger than old y-component

Also: u-axis has been rotated away from P, as compared to x-axis

$\Rightarrow$ u-component must be smaller than old x-component

Allows you to eliminate some wrong answers!

Now that $\vec{r}_u$ is known, we have two choices

1. more trig: $\vec{r}_v = |\vec{r}|\sin 53.1° = (5\text{cm})\left(\frac{4}{5}\right) = \boxed{4.00\text{cm}}$

2. Geometry / Pythagorean theorem: $|\vec{r}|^2 = |\vec{r}_v|^2 + |\vec{r}_u|^2$

$\Rightarrow |\vec{r}_v| = \sqrt{|\vec{r}|^2 - |\vec{r}_u|^2} = \sqrt{(5\text{cm})^2 - (3\text{cm})^2} = \boxed{4.00\text{cm}}$