Physics 2211 A/B
Fall 2015
Exam 1

Test form 611

Name Official Solutions

Recitation Section (see back of test):__________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

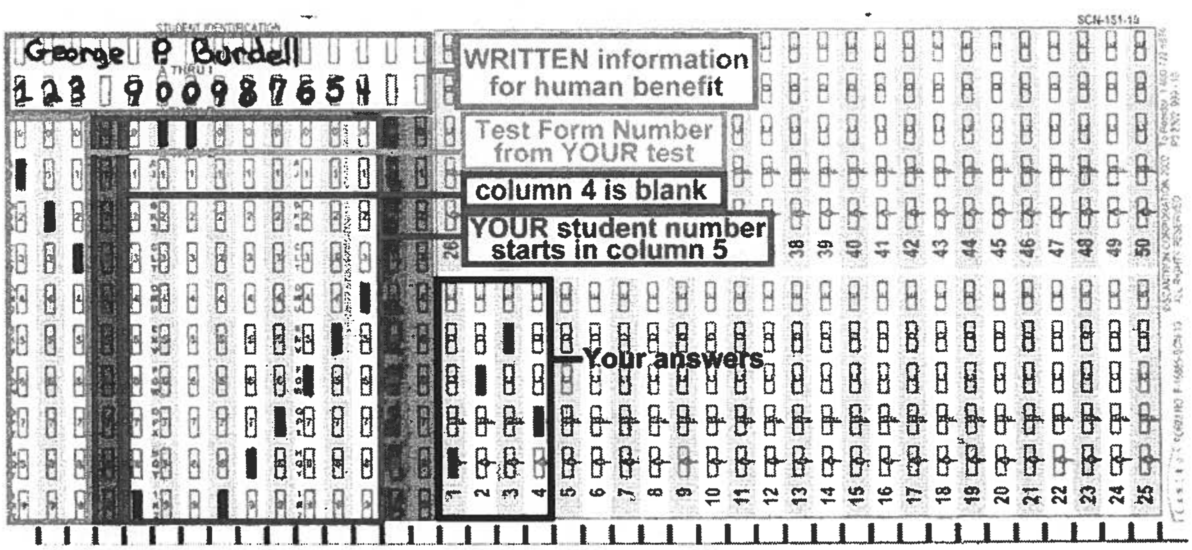
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

Your test form is: 611



Our next test will be on Monday, October 05!

The following problem will be hand-graded. <u>*Show all your work for this problem.*</u> *Make no marks and leave no space on your answer card for it.*

[I] *(20 points)* The tortoise and the hare are in a footrace, covering a straight-line distance D. The tortoise maintains a steady, plodding speed v_o throughout the race. The hare starts out at a rapid pace $5v_o$, but stops to complacently take a nap after completing ¼ of the distance. He awakens to see the tortoise still plodding along, now 90% finished. In a panic, the hare sets off at his maximum speed, $6v_o$.

Who wins the race? By what time interval Δt does the winner beat the loser?

Express your answer for Δt algebraically in terms of D and v_o.

① Hare stops after $\vec{\Delta X}_{H1} = \langle +D/4 \rangle \rightarrow$ must displace $\vec{\Delta X}_{H2} = \langle +3D/4 \rangle$ to finish race

② Tortoise at $\vec{x} = \langle +9D/10 \rangle \rightarrow$ must displace $\vec{\Delta X}_{T2} = \langle +D/10 \rangle$ to finish race

- time to finish race: $\vec{\Delta X}_{T2} = \langle +V_0 \rangle \Delta t_{T2} \longrightarrow +D/10 = V_0 \Delta t_{T2}$

$$\rightarrow \boxed{\Delta t_{T2} = \frac{D}{10V_0}}$$

- meanwhile, Hare must displace $\langle +3/4 D \rangle$ at speed $6V_0$:

$$\langle +3D/4 \rangle = \vec{\Delta X}_{H2} = \langle +6V_0 \rangle \Delta t_{H2} \rightarrow \boxed{\Delta t_{H2} = \frac{3D}{4}\frac{1}{6V_0} = \frac{D}{8V_0}}$$

since $\frac{D}{10V_0} < \frac{D}{8V_0}$, $\boxed{\text{tortoise wins race}}$

$$\text{time lag } \Delta t = \Delta t_{loser} - \Delta t_{winner} = \frac{D}{8V_0} - \frac{D}{10V_0}$$

$$= \frac{D}{V_0}\left[\frac{1}{8} - \frac{1}{10}\right] = \frac{D}{V_0}\left[\frac{5}{40} - \frac{4}{40}\right]$$

$$\boxed{\Delta t_{lag} = \frac{D}{40V_0}}$$

note units of expression:

$D \rightarrow$ [length]

$V_0 \rightarrow$ [length/time]

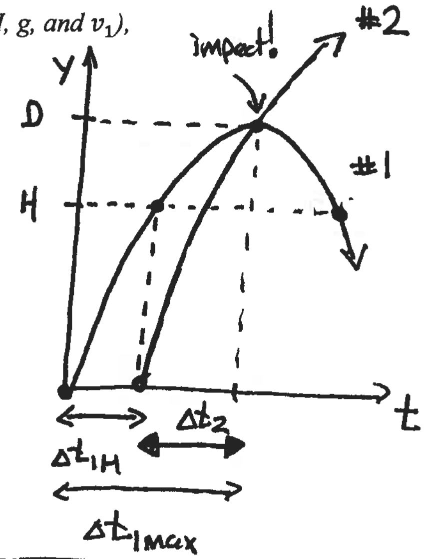
so $\frac{D}{V_0} \rightarrow \left[\frac{\text{length}}{\text{length/time}}\right] = [\text{time}]$

The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

[II] *(20 points)* During a manned mission to Mars, Astronaut Mark Watney is stranded on the Red Planet (where the local acceleration is $g = 3.71$ m/s^2). In frustration, he throws a stone upward with initial speed $v_1 = 22.2$ m/s, then reaches down to pick up and throw a second stone upward. If he releases the second stone at a moment when the first stone has reached a height $H = 44.4$ m, with what minimum speed v_2 must he throw the second stone so that it will strike the first stone *before* it reaches its maximum height?

Your grader is likely to award more credit if you work algebraically (using the symbols H, g, and v_1), saving numerical calculations only for intermediate and/or final values...

Don't forget to use the Mars value for g in your calculation!



Let Δt_{1H} = time for #1 to rise to height H

Δt_{1max} = " " " " " " maximum height: $\vec{Y}_{max} = \langle +D \rangle$

Since Δt_{1H} also = time lag before throwing #2,
rise time for #2 is $\Delta t_2 = \Delta t_{1max} - \Delta t_{1H}$

① Rise time — #1 to max height

$\vec{V}_{yi} = \langle +V_0 \rangle$, $\vec{V}_{yf} = \langle 0 \rangle$ at apex

$\Delta\vec{V}_1 = \langle -g \rangle \Delta t_1 \rightarrow \langle -V_0 \rangle = \langle -g \rangle \Delta t_{1max}$

$$\boxed{\Delta t_{1max} = \frac{V_0}{g} = 5.98\text{sec}}$$

② Height at apex — best found from speed equation:

$V_f^2 = V_i^2 + 2\langle -g \rangle \langle \Delta y \rangle \longrightarrow 0 = V_0^2 - 2gD \rightarrow \boxed{D = \frac{V_0^2}{2g} = 66.4\text{m}}$

③ Rise time to height H:

$\vec{\Delta y} = \vec{V}_{iy}\Delta t + \frac{1}{2}\langle -g \rangle \Delta t^2 \longrightarrow \langle +H \rangle = \langle +V_0 \rangle \Delta t_{1H} + \frac{1}{2}\langle -g \rangle \Delta t_{1H}^2$

$\frac{1}{2} g \Delta t_{1H}^2 - V_0 \Delta t_{1H} + H = 0 \longrightarrow$ quadratic in Δt_{1H}

roots: $\Delta t_{1H} = \dfrac{+V_0 \pm \sqrt{V_0^2 - 2gH}}{g}$ → choose smaller root (neg sign)

$\Delta t_{1H} = \dfrac{V_0}{g} - \dfrac{\sqrt{V_0^2 - 2gH}}{g} = 5.98\text{sec} - 3.45\text{sec}$

$\Delta t_{1H} = 2.54\text{sec}$

④ Rise time for #2 is

$\Delta t_2 = \Delta t_{1max} - \Delta t_{1H} = \dfrac{V_0}{g} - \left(\dfrac{V_0}{g} - \dfrac{\sqrt{V_0^2 - 2gH}}{g}\right) \boxed{= 3.45\text{sec} = \Delta t_2}$

⑤ Require #2 to also be at $\vec{Y}_{max} = \langle +D \rangle$:

$\vec{\Delta y} = \vec{V}_{iy}\Delta t + \frac{1}{2}\langle -g \rangle \Delta t^2 \longrightarrow \langle +D \rangle = \langle +V_2 \rangle \Delta t_2 + \frac{1}{2}\langle -g \rangle \Delta t_2^2$

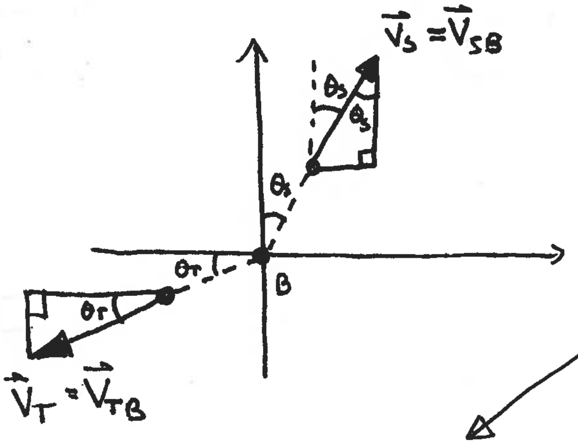
$$\boxed{V_2 = \frac{D + \frac{1}{2} g \Delta t_2^2}{\Delta t_2} = 25.7\text{m/s}}$$

The following problem will be hand-graded. *<u>Show all your work for this problem</u>.* *Make no marks and leave no space on your answer card for it.*

[III] *(20 points)* Two ships depart from the island of Bermuda at noon. Ship S travels in a direction $\theta_S = 17.0°$ east of north with a speed $v_S = 22.9$ knots. Ship T travels in a direction $\theta_T = 23.5°$ south of west with a speed $v_T = 19.3$ knots. As the sun sets at 7:00pm, what is the velocity of Ship T relative to Ship S?

Your work will be graded on the quality of your vector diagram, as well as the accuracy of your answer!

You are expected to use the process of vector decomposition for this problem; use of the "Law of Cosines" will be penalized as an error in basic physics.



note that positions, and elapsed times are unimportant

velocities are constant, so their relative velocity is also a constant

$$\boxed{\text{Require: } \vec{V}_{TB} = \vec{V}_{TS} + \vec{V}_{SB}}$$

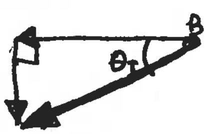
$\vec{V}_{TB}$: ship T to Barbados

$\vec{V}_{SB}$: ship S to Barbados

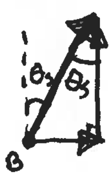
$\vec{V}_{TS}$: Ship T to Ship S — this vector is our objective!

rearranging: $\boxed{\vec{V}_{TS} = \vec{V}_{TB} - \vec{V}_{SB}}$ Perform this subtraction

where



$$\vec{V}_{TB} = \langle -V_T \cos\theta_T \rangle \hat{i} + \langle -V_T \sin\theta_T \rangle \hat{j} = \langle -17.7\,kt \rangle \hat{i} + \langle -7.70\,kt \rangle \hat{j}$$

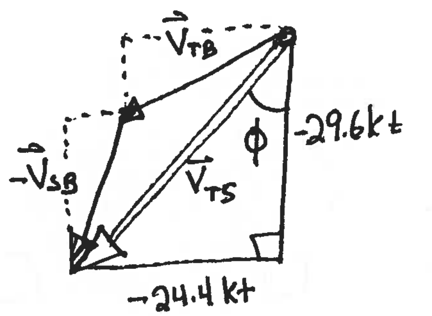


$$\vec{V}_{SB} = \langle +V_S \sin\theta_S \rangle \hat{i} + \langle +V_S \cos\theta_S \rangle \hat{j} = \langle +6.70\,kt \rangle \hat{i} + \langle +21.9\,kt \rangle \hat{j}$$

Subtraction = "Addition of $-\vec{V}_{SB}$"

Subtract:

$$\vec{V}_{TB} = \langle -17.7 - (+6.70) \rangle kt\, \hat{i} + \langle -7.7 - (+21.9) \rangle kt\, \hat{j}$$



Cartesian form is an acceptable answer

$$\boxed{\vec{V}_{TB} = \langle -24.4\,kt \rangle \hat{i} + \langle -29.6\,kt \rangle \hat{j}}$$

Using some trig: $|\vec{V}_{TB}| = \sqrt{V_x^2 + V_y^2} = 38.4$ knots

$$\phi = \tan^{-1}\left(\frac{|V_x|}{|V_y|}\right) = 39.5°$$

so $\boxed{\vec{V}_{TB} = 38.4 \text{ knots, directed } 39.5° \text{ West of South}}$ (or 50.5° south of west)

The next two questions involve the following situation:

A wind-up spring-powered toy car begins at rest at the origin. It moves with a time-dependent acceleration, given by the expression

$$\vec{a}(t) = \langle +\tfrac{a_o}{T}\, t \rangle, \quad \text{for } 0 \le t \le T$$

Here, a_o and T are constants having units of accleration and time, respectively. At time T, the spring winds down, and the car begins to coast with a constant velocity.

Question value 5 points

(1) What is the average velocity of the car between times $t = 0$ and $t = T$?

$\boxed{\text{(a)} \quad \vec{v}_{av} = \langle +\tfrac{a_o T}{6} \rangle}$

* (b) $\vec{v}_{av} = \langle +\tfrac{a_o T}{2} \rangle$

(c) $\vec{v}_{av} = \langle +\tfrac{a_o}{T} \rangle$

* (d) $\vec{v}_{av} = \langle +\tfrac{a_o T}{4} \rangle$

(e) $\vec{v}_{av} = \langle +\tfrac{a_o}{3T} \rangle$

* Correct physical units for a velocity

average velocity is defined as $\vec{V}_{av} = \frac{\overrightarrow{\Delta x}}{\Delta t} = \frac{\vec{X}_f - \vec{X}_i}{\Delta t}$

→ need to know initial ($\vec{X}_i = 0$) and final positions

• Find $x(t)$

First: $\vec{V}(t) = \int \vec{a}(t)dt + \vec{C} = \langle + \frac{a_o}{T}\frac{t^2}{2} \rangle + \vec{C}$

$\vec{V}(0) = 0$ implies $\vec{C} \equiv 0$

so $\boxed{\vec{V}(t) = \langle + \frac{a_o}{2T} t^2 \rangle} \rightarrow \vec{X}(t) = \int \vec{V}(t)dt + \vec{C} = \langle \frac{a_o}{2T}\frac{t^3}{3} \rangle + \vec{C}$

$\vec{X}(0) = 0$ implies $\vec{C} \equiv 0$

$\vec{X}(t) = \langle + \frac{a_o}{6T} t^3 \rangle \rightarrow \vec{X}_f = \vec{X}(T) = \langle + \frac{a_o T^2}{6} \rangle$

then $\vec{V}_{av} = \dfrac{\langle + \frac{a_o T^2}{6} \rangle - \langle 0 \rangle}{T}$

$$\boxed{\vec{V}_{av} = \langle + \frac{a_o T}{6} \rangle}$$

Question value 5 points

(2) What is the final coasting velocity of the car after the spring winds down?

* (a) $\vec{v}_f = \langle +\tfrac{a_o T}{4} \rangle$

(b) $\vec{v}_f = \langle +\tfrac{a_o}{3} \rangle$

(c) $\vec{v}_f = \langle +\tfrac{a_o}{2T} \rangle$

(d) $\vec{v}_f = \langle +\tfrac{a_o}{T^2} \rangle$

$\boxed{\text{(e)} \quad \vec{v}_f = \langle +\tfrac{a_o T}{2} \rangle}$

$\vec{V}_f = \vec{V}(t = T)$

from above, $\vec{V}(t) = \langle + \frac{a_o}{2T} t^2 \rangle$

so $\boxed{\vec{V}_f = \langle + \frac{a_o T}{2} \rangle}$

* Correct physical units for a velocity:

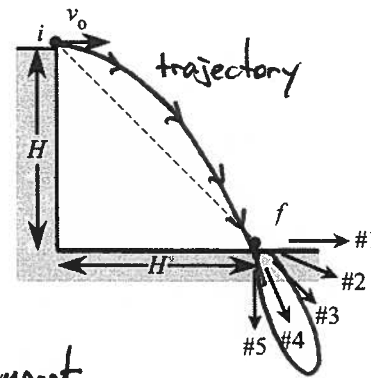
$[a_o] \Rightarrow$ [length/time^2]

$[T] \Rightarrow$ [Time]

so $[a_o T] \rightarrow [(\text{length}/\text{time}^2)(\text{time})] = [\text{length}/\text{time}] = [\text{velocity}]$

The next two questions involve the following situation:

A cannonball is fired horizontally with an initial speed v_o, from atop a cliff of height H. The cannonball is observed to land a horizontal distance from the base of the cliff that is coincidentally exactly equal to H.



Question value 5 points

(3) Which of the arrows in the figure best depicts the velocity of the cannonball at impact?

(a) #4
(b) #2
(c) #5
(d) #1
(e) #3

Ball follows curved trajectory, with velocity vector tangent to curve

For sketched trajectory, tangent line at impact is steeper than the 45° straight line

$\Rightarrow$ $\boxed{\text{Vector \#4}}$ is the best fit, steeper than 45°

Question value 5 points

(4) What is the specific value of v_o that will result in the trajectory described?

(a) $v_o = \sqrt{H/2g}$
(b) $v_o = \sqrt{gH/2}$
* (c) $v_o = \sqrt{2gH}$
(d) $v_o = \sqrt{2H/g}$
(e) $v_o = \sqrt{gH}$

* Correct physical units for a velocity or speed

① time to reach ground : vertical position equation:

$$\Delta y = \underbrace{\cancel{V_{oy}}}_{\text{zero}}\Delta t + \tfrac{1}{2}\langle -g\rangle \Delta t^2$$

$$\langle -H\rangle = \tfrac{1}{2}\langle -g\rangle \Delta t^2 \rightarrow \Delta t = \sqrt{\frac{2H}{g}}$$

② Plug result into horizontal motion equation, where $\vec{V}_x = \langle +V_0\rangle$ = constant :

$$\langle +H\rangle = \langle +V_0\rangle \Delta t$$

$$V_0 = \frac{H}{\Delta t} = \frac{H}{\sqrt{2H/g}} = H\sqrt{\frac{g}{2H}} = \boxed{\sqrt{\frac{gH}{2}}}$$

Question value 5 points

(5) Fill in the blanks: A motion diagram depicts an object that is slowing down. In the diagram, the ________ between successive frames must be __________ in size.

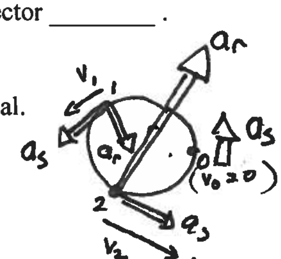
slowing down:

spacings (aka $\boxed{\text{displacements}}$) get $\boxed{\text{smaller}}$

(a) displacements ; decreasing
(b) ~~time intervals ; increasing~~
(c) displacements ; increasing
(d) positions ; decreasing
(e) ~~time intervals ; decreasing~~

Question value 5 points

(6) Fill in the blanks: A car on a circular track begins at rest and experiences a constant angular acceleration around the track. As it moves, the magnitude of it's acceleration vector _________ and the direction of its acceleration vector ________ .



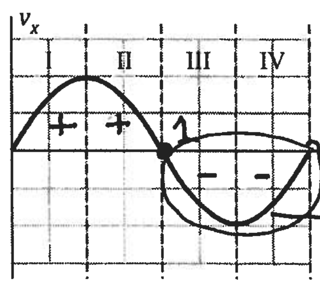
(a) remains constant ; is always tangential
(b) decreases at first, but then increases ; changes from being entirely radial to being entirely tangential.
(c) remains constant ; maintains a fixed orientation somewhere *between* tangential and radial
(d) increases ; is always radial
(e) increases ; changes from being entirely tangential to mostly radial

initial $V_s = 0$ so $a_{radial} = V_s^2/R = 0$ But $a_s \neq 0$

later: same a_s, but now $V_s \neq 0$ so $a_r \neq 0$ → $|\vec{a}|$ is larger, and $\vec{a}$ has changed from all tangential

Question value 5 points

(7) The plot at right depicts the velocity-versus-time graph for a particle that stars at rest at the origin. During which interval(s) is the particle moving toward the origin?



Moving Toward origin: positive position, and negative velocity

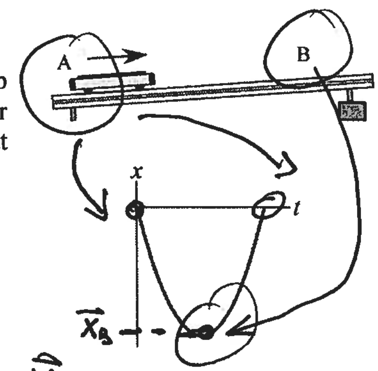
Value for $\vec{V}$ is negative

(a) During interval II only.
(b) During intervals III and IV.
(c) During none of the intervals shown.
(d) During intervals II and IV.
(e) During intervals II and III.

during I+II, particle has positive displacement
→ at time 1, $\vec{X}_1 > 0$
after time 1, $\vec{V} < 0$
} toward origin

Question value 5 points

(8) A cart is given an initial shove up an inclined ramp (top). The cart starts at A, coasts up the track and stops at B, and then returns back down the track to A. An observer collects position data and constructs the *x*-vs-*t* graph shown at right (bottom). What coordinate system was the observer using to collect data?



(a) The coordinate system that was used cannot be inferred from the graph.
(b) A system with the ~~origin at B~~ and the positive direction pointing *downslope*.
(c) A system with the origin at A and the positive direction pointing *downslope*.
(d) A system with the origin at B and the positive direction ~~*pointing upslope*~~.
(e) A system with the origin at A and the positive direction ~~*pointing upslope*~~.

$\vec{X}_i = 0$
$\vec{X}_f = 0$

X_{stop} is negative

$\vec{X}_A = \vec{X}_i = 0$, so $\boxed{\text{origin is at A}}$

$\vec{X}_B$ is clearly negative, so upslope = neg

$\boxed{\text{downslope} = \text{pos}}$