Physics 2211 A/B
Fall 2015
Exam 3

Test form 633

Name Solution Set

Recitation Section (see back of test):_________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

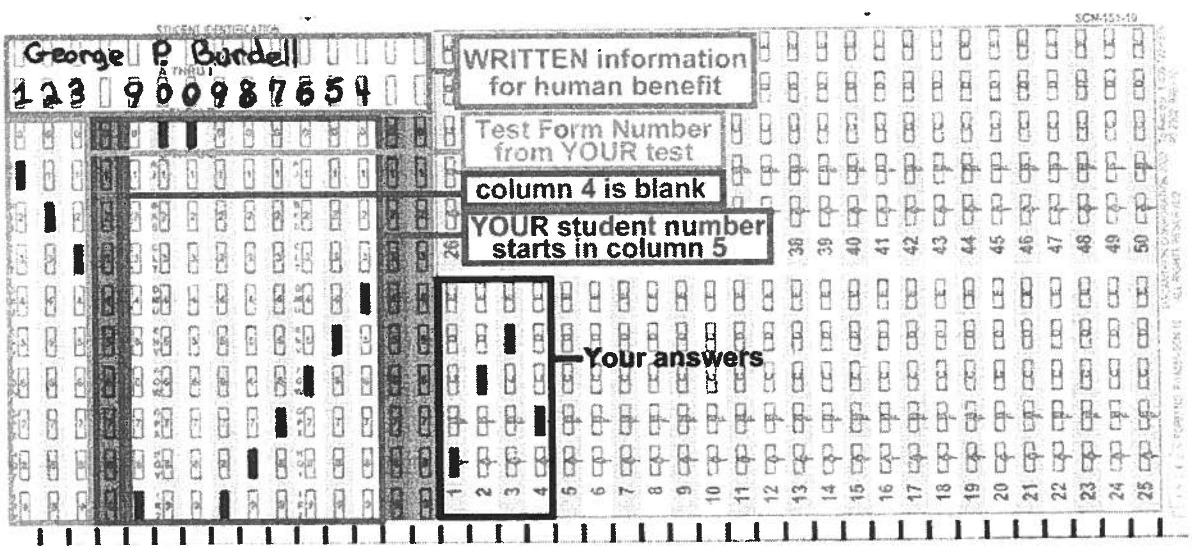
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

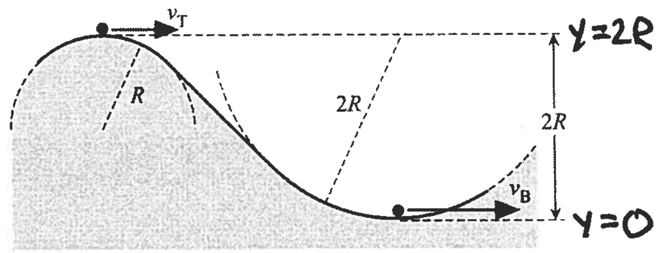
Your test form is: 633



Our next test will be on Monday, November 23!

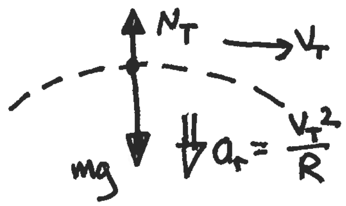
The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[I] *(20 points)* A roller-coaster car is travelling along a frictionless track. As it passes over the top of a hill having radius of curvature R, it has just enough speed v_T to become momentarily airborne. It than travels down into a valley; at the very bottom of the valley (a distance $2R$ below the top of the hill), the radius of curvature is $2R$.



What will be the percieved weight of a passenger in the roller-coaster car, at the very bottom of the hill? Express your answer as a multiple of the passenger's true weight, mg.

① At top of hill, circular motion



→ "momentarily airborne" means $N_T \to 0$ at very top

SO: $\sum \vec{F}_r = m\vec{a}_r \to \langle -\underbrace{N_T}_{\text{zero}} \rangle + \langle +mg \rangle = m \langle +v_T^2/R \rangle$

$\to \boxed{v_T^2 = gR}$

② Conservation of energy from top of hill to bottom of valley

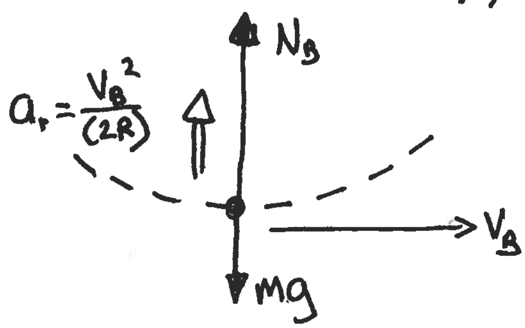
- choose $y=0$ and $U_{gB}=0$ at bottom of hill

→ at top of hill, $y=2R$ and $U_{gT} = 2mgR$

$K_T + U_T = K_B + U_B \longrightarrow \frac{1}{2}mv_T^2 + 2mgR = \frac{1}{2}mv_B^2 + 0$

$\frac{1}{2}m(gR) + 2mgR = \frac{1}{2}mv_B^2 \to \boxed{v_B^2 = 5gR}$

③ At bottom of valley, circular motion



$\sum \vec{F}_r = m\vec{a}_r \to \langle +N_B \rangle + \langle -mg \rangle = m \langle + \frac{v_B^2}{2R} \rangle$

$N_B = mg + m\frac{v_B^2}{2R} = mg + m\left(\frac{5gR}{2R}\right) = mg + \frac{5}{2}mg$

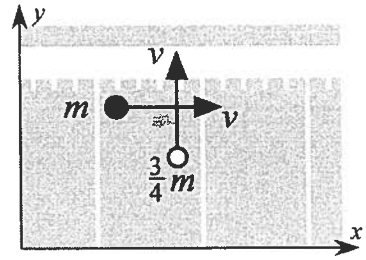
$\boxed{N_B = \frac{7}{2}mg}$

Normal force by seat of car provides passenger's sense of "perceived weight"

The following problem will be hand-graded. <u>*Show all your work for this problem.*</u> *Make no marks and leave no space on your answer card for it.*

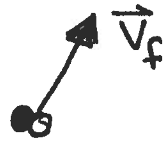
[II] *(20 points)* A football player (mass m) is returning a punt, running parallel to the sideline at speed v. The runner is tackled by the kicker (mass $\frac{3}{4}m$), who collides with him moving directly perpendicular to the sideline at the same speed v. The kicker "wraps up", and the two players fall to the ground together.

What impulse is delivered to the runner during the collision? Express your answer in cartesian vector form, in terms of the parameters m and v.



"Wraps up" implies kicker grabs runner, and they fall together to the ground

↳ collision: just before (above) : just after



(players are moving together)

⇒ perfectly inelastic collision

$$\vec{P}_i = \vec{P}_f$$

$$m\langle +v\rangle\hat{i} + \tfrac{3}{4}m\langle +v\rangle\hat{j} = (m+\tfrac{3}{4}m)\,\vec{V}_f = \tfrac{7}{4}m\vec{V}_f$$

so
$$\vec{V}_f = \tfrac{4}{7}\left[+v\,\hat{i} + \tfrac{3}{4}v\,\hat{j}\right] = \boxed{\langle +\tfrac{4}{7}v\rangle\hat{i} + \langle +\tfrac{3}{7}v\rangle\hat{j}}$$

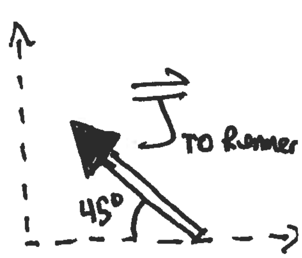
"Final" velocity of both players after they collide, but before they hit the ground

Now: "impulse to runner"

Use
$$\vec{J}_R = \Delta\vec{P}_R = m\Delta\vec{V}_R = m\vec{V}_f - m\vec{V}_{R,i}$$

$$= m\left[\left(\tfrac{4}{7}v\,\hat{i} + \tfrac{3}{7}v\,\hat{j}\right) - \left(v\,\hat{i}\right)\right]$$

$$\Rightarrow \boxed{\vec{J}_R = mv\left[-\tfrac{3}{7}\hat{i} + \tfrac{3}{7}\hat{j}\right]}$$



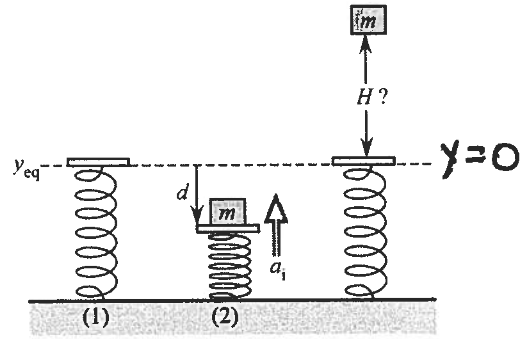
$$= \tfrac{3\sqrt{2}}{7}mv\left[-\tfrac{1}{\sqrt{2}}\hat{i} + \tfrac{1}{\sqrt{2}}\hat{j}\right]$$

$$= \tfrac{3\sqrt{2}}{7}mv$$ @ 45° angle backwards and toward the sideline

The following problem will be hand-graded. *<u>Show all your work for this problem</u>.* *Make no marks and leave no space on your answer card for it.*

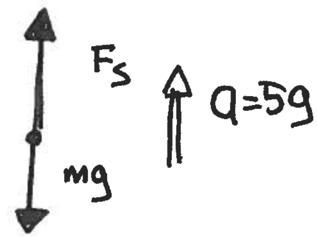
[III] *(20 points)* A vertical spring has a massless platform attached to it (1). A block of mass m is placed of the platform, and then the spring is compressed down by a distance d below its natural length (2). The spring is then released, and the block is launched vertically up into the air. An accelerometer indicates that at the moment of release, the block experiences an initial upward acceleration $\vec{a}_i$ of magnitude $5g$.

How high H will the block rise above the spring's natural equilibrium height? Express your answer as a multiple of the original compression distance d.



① At moment of release, $\vec{a}_y = \langle +5g \rangle$

→ forces acting are spring and gravity



$$\langle +F_s \rangle + \langle -mg \rangle = m\langle +5g \rangle$$

$$-k\Delta y = 6mg$$

$$-k(-d) = 6mg \longrightarrow \boxed{k = \frac{6mg}{d}}$$

use this knowledge of the elastic constant in an energy problem

② Conservation of energy

- choose both $U_g = 0$ and $U_s = 0$ at equilibrium position of spring → set $y_{eq} \equiv 0$

Just prior to release, compressed by distance d:

$$U_g = -mgd \quad U_s = \tfrac{1}{2}kd^2 \quad K_i = 0$$

At highest point: $U_g = +mgH$, $U_s = 0$, $K_f = 0$

So $$+mgH + 0 + 0 = -mgd + \tfrac{1}{2}kd^2 + 0$$

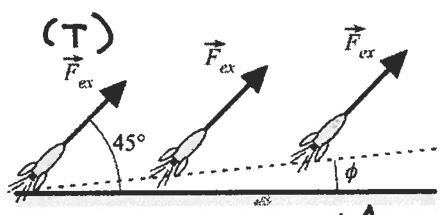
$$mgH = -mgd + \tfrac{1}{2}\left(\frac{6mg}{d}\right)d^2 = -mgd + 3mgd = +2mgd$$

$$\Rightarrow \boxed{H = 2d}$$

F_{ex}, or just T

Question value 5 points

(1) A toy rocket of mass m is lauched from the ground, with it's nose maintaining a gyroscopically controlled angle $\theta = 45°$ with above the horizontal. It's trajectory as it leaves the ground, however, is a straight-line path directed at an angle $\phi = 15°$ above the horizontal. What is the magnitude of the thrust force generated by the rocket, as a multiple of the rocket's weight?

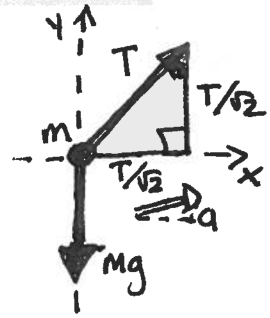


- ~~(a) $\vec{F}_{ex} = 0.73\ mg$~~
- $\boxed{\text{(b) } \vec{F}_{ex} = 1.93\ mg}$
- (c) $\vec{F}_{ex} = 1.41\ mg$
- ~~(d) $\vec{F}_{ex} = 1.37\ mg$~~
- (e) $\vec{F}_{ex} = 3.73\ mg$

We saw in class that to lift off, thrust must be greater than $\sqrt{2}\,mg = 1.414\,mg$!

direction of travel:

$$\tan\phi = \frac{V_y}{V_x} = \frac{a_y \Delta t}{a_x \Delta t} = \frac{\Sigma F_y/m}{\Sigma F_x/m} = \frac{+\frac{1}{\sqrt{2}}T - mg}{\frac{1}{\sqrt{2}}T}$$



$$\text{so } \frac{1}{\sqrt{2}}T\tan\phi = \frac{1}{\sqrt{2}}T - mg$$

$$\sqrt{2}\,mg = T(1-\tan\phi)$$

$$T = \frac{\sqrt{2}}{1-\tan\phi}\,mg = \frac{1.414}{0.732}\,mg \doteq \boxed{1.93\,mg}$$

Question value 5 points

(2) Spring A has elastic constant k, and spring B has elastic constant $2k$. Spring B is stretched by a distance d, while spring A is compressed by a distance $2d$. Compare the potential energy stored in the two springs.

- $\boxed{\text{(a) } U_A = 2U_B}$
- (b) $U_A = U_B/2$
- ~~(c) $U_A = -2U_B$~~
- (d) $U_A = U_B$
- ~~(e) $U_A = -U_B/2$~~

spring energy is never negative!

$$U_B = \frac{1}{2}(2k)(d)^2 = kd^2$$

$$U_A = \frac{1}{2}(k)(-2d)^2 = 2kd^2$$

$$\boxed{U_A = 2U_B}$$

Question value 5 points

(3) A moving block of mass $2m$ strikes with a second block of mass m that is initially stationary, resulting in a perfectly inelastic 1D collision. What percentage of the system's initial energy is lost in the collision?

→ same final velocity, so

$$\vec{P}_i = \vec{P}_f \longrightarrow 2m\langle +V\rangle = 3m\vec{V}_f \rightarrow \vec{V}_f = \frac{2}{3}V\,\hat{i}$$

- ~~(a) 0%~~ this would be perfectly elastic
- (b) 50%
- ~~(c) 100%~~ nonsense!
- (d) 67%
- $\boxed{\text{(e) } 33\%}$

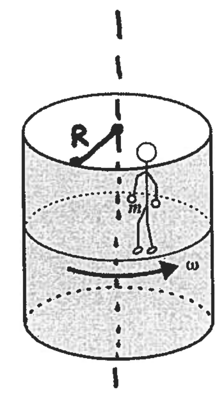
KE before collision: $\frac{1}{2}(2m)V^2 = mV^2 = K_i$

total KE afterward: $\frac{1}{2}(3m)\left(\frac{2}{3}V\right)^2 = \frac{2}{3}mV^2 = K_f$

KE loss was therefore $|K_f - K_i| = \frac{1}{3}mV^2 = \frac{1}{3}K_i$

→ $\boxed{\text{33\% of initial energy was lost}}$

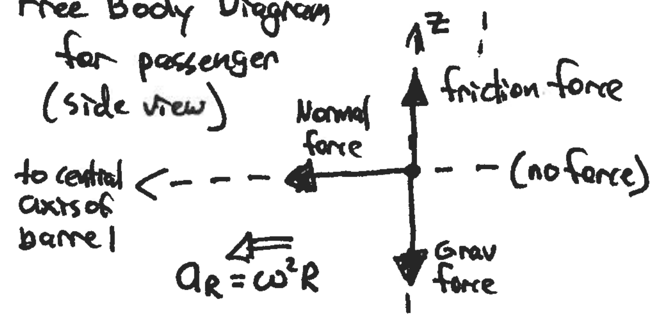
The next two questions involve the following situation:
In the "Barrel Ride" at the amusement park, a passenger finds himself "stuck" to the inner wall of a vertical cylinder that is rotated at high speed, at which point the floor drops out from under his feet.



Question value 5 points
(4) What is the force that pushes the passenger radially outward so that he will stick to the wall?

- $\boxed{\text{(a) There is no force at all pushing outward on the passenger.}}$
- (b) The force of the passenger's inertia.
- (c) The centripetal force by the barrel wall.
- (d) The static friction force by the barrel wall.
- (e) The normal force by the barrel wall.

Free Body Diagram for passenger (side view)



Question value 5 points
(5) The ride is designed to spin the barrel at a minimum speed ω, before allowing the floor to drop. Suppose that on a rainy day, the walls of the barrel become slippery, so the the coefficient of static friction is half of its usual value. With what mimimum speed must the barrel rotate, on the rainy day?

- ~~(a) 0.70 ω~~
- $\boxed{\text{(b) 1.4 }\omega}$
- (c) 2.0 ω
- (d) 4.0 ω
- ~~(e) 0.50 ω~~

ω must be larger on a rainy day!

Using diagram above: $\sum \vec{F}_z = 0 \rightarrow \langle +f_s \rangle + \langle -mg \rangle = 0$

$\rightarrow$ actual friction is $f_s = mg$

$\sum \vec{F}_r = m\vec{a}_r$

$\rightarrow \langle +N \rangle = m \langle +\omega^2 R \rangle$

$\rightarrow$ maximum possible friction is $f_{s,max} = \mu_s N = \mu_s m\omega^2 R$

actual friction $\leq$ maximum possible friction

$f_s \leq f_{s,max} \rightarrow \cancel{m}g \leq \mu_s \cancel{m}\omega^2 R$

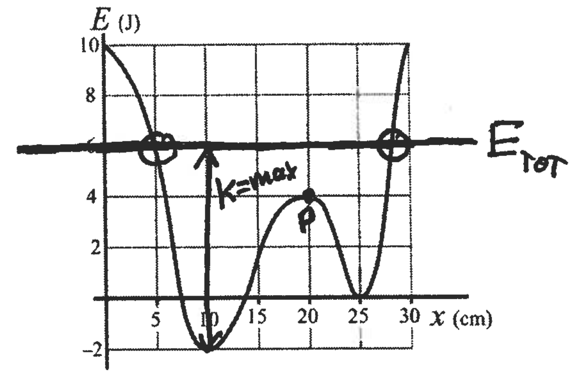
$\omega^2 \geq \frac{g}{\mu_s R} \qquad \omega \geq \sqrt{\frac{g}{\mu_s R}}$

$\rightarrow$ Use minimum value $\quad \omega_{min} = \sqrt{\frac{g}{\mu_s R}}$

on a rainy day, let $\mu \rightarrow \frac{1}{2}\mu \quad \omega_{rain} = \sqrt{\frac{g}{\mu_s/2\, R}} = \sqrt{\frac{2g}{\mu_s R}} = \sqrt{2}\,\omega_{min}$

The next two question involve the following situation:

An object of mass $m = 1.0$ kg moves while subject to the potential energy diagram shown below. The object is observed to have a speed $v = 2.0$ m/s when the ball is at location $x = 20$ cm. position P



Question value 5 points

(6) What will be the turning points of the object's motion along the x-axis?

(a) $x = 6$ cm and $x = 27$ cm
(b) $x = 0$ cm and $x = 30$ cm
(c) $\boxed{x = 5 \text{ cm and } x = 28 \text{ cm}}$
(d) It will not have *any* turning points in this situation.
(e) $x = 15$ cm and $x = 23$ cm

Note K at point P $= \frac{1}{2}mv^2 = 2\,J$
U " " " $= 4\,J$

$\boxed{E_{TOT} = 6\,J} \rightarrow$ turning points occur where $U = E_{TOT}$, at $\boxed{x = 5\text{cm}, x \approx 28\text{cm}}$

Question value 5 points

(7) What maximum speed can the object have, during its motion?

(a) 2.8 m/s
(b) 2.0 m/s
(c) ~~1.4 m/s~~
(d) 3.5 m/s
(e) $\boxed{4.0 \text{ m/s}}$

absurd choice! we already know $V = 2.0$m/s at P!!

Max speed at location where $U = \min = -2\,J$

$K_{max} + U = E_{TOT}$

$\rightarrow K_{max} = E_{TOT} - U = (6J) - (-2J) = 8\,J$

so $\frac{1}{2}mV_{max}^2 = 8\,J \rightarrow V_{max} = \sqrt{\frac{2(8J)}{(1\,kg)}} = \boxed{4.0\,m/s}$

Question value 10 points

(8) A rubber ball and an egg having <u>identical</u> masses are dropped from a height h. The ball rebounds to height h, while the eggs splatters on the ground without rebounding. The ball is in contact with the ground for a total time T_0, while the duration of the egg's splatter is $2T_0$. Compare the average force magnitude felt by each object, while in contact with the ground.

(a) $F_{ball} = (1/2)\, F_{egg}$
(b) $F_{ball} = F_{egg}$
(c) $\boxed{F_{ball} = 4\, F_{egg}}$
(d) $F_{ball} = 2\, F_{egg}$
(e) $F_{ball} = (1/4)\, F_{egg}$

$$|\vec{F}_{av}| = \frac{|\vec{J}|}{\Delta t} = \frac{|m\Delta\vec{V}|}{\Delta t}$$

• assume <u>each</u> strikes ground with speed V ← free fall to ground

$$\Delta\vec{V}_{egg} = \langle 0 \rangle - \langle -V \rangle = \langle +V \rangle$$

$$\Delta\vec{V}_{ball} = \langle +V \rangle - \langle -V \rangle = \langle +2V \rangle$$

$$F_{ball} = \frac{m|\Delta\vec{V}_{ball}|}{\Delta t_{ball}} = \frac{m \cdot 2V}{T_0} = \frac{2mV}{T_0}$$

$$F_{egg} = \frac{m|\Delta\vec{V}_{egg}|}{\Delta t_{egg}} = \frac{m \cdot V}{2T_0} \rightarrow F_{ball} = 4\, F_{egg}$$