Physics 2211 A/B
Fall 2015
**Exam 4**

Test form **642**

Name Solution Set

Recitation Section (see back of test): ________

**A**

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

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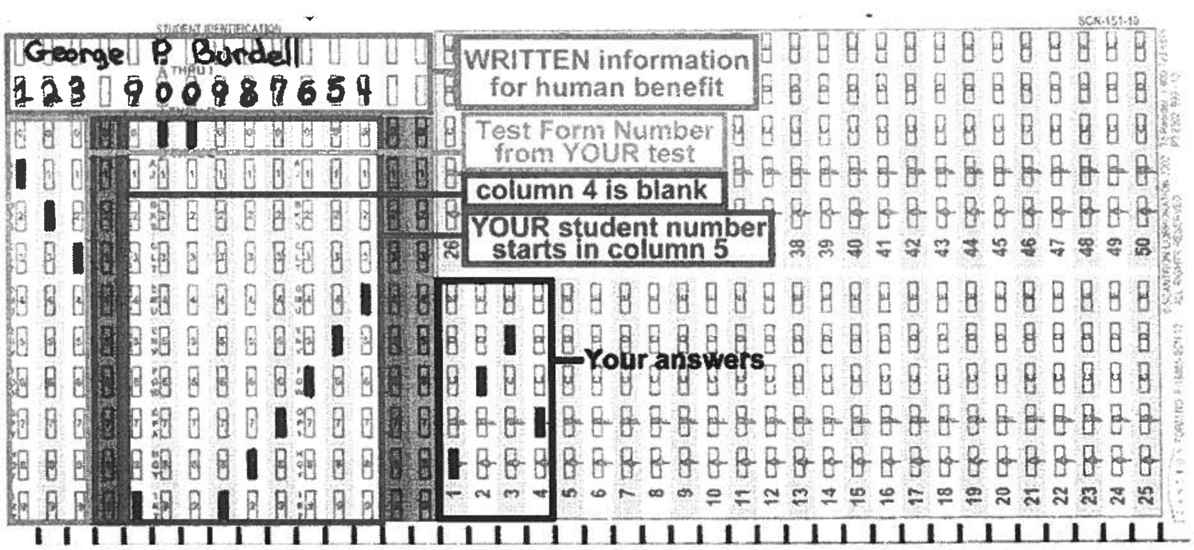
**Your test form is: 642**

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## Our Final Exam will be held on Monday, December 7 at 8:00am!

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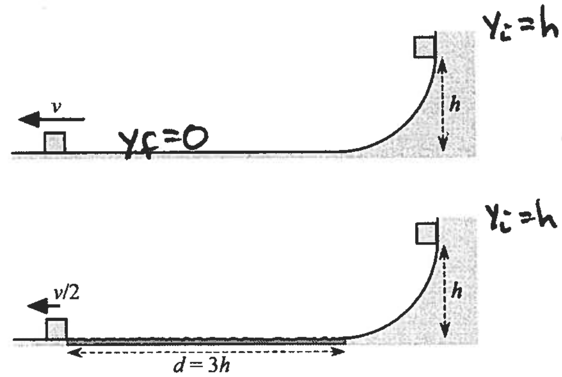
*The following problem will be hand-graded.* *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

**[I]** *(20 points)* A block is released from rest on a ramp of height $h$. It slides without friction to the bottom of the ramp, and then continues across a horizontal frictionless surface, with a final speed $v$. A second identical block is released from the same height on a similar ramp. When it reaches the bottom of the ramp, it crosses a rough horizontal patch having length $d = 3h$. When it leaves the rough patch, it has a speed $v/2$.

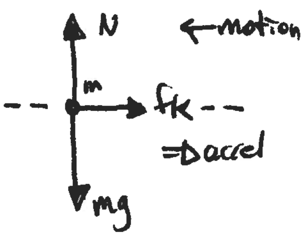
Use the General Work-Energy Principle to find the coefficient of friction of the rough patch.




1st Block – energy is conserved: $K_i + U_i = K_f + U_f$

$0 + mgh = \frac{1}{2}mv^2 + 0 \longrightarrow \boxed{\frac{1}{2}mv^2 = mgh}$

2nd Block – Work-Energy principle says $W_{nc} = \Delta E$




- work by friction is $W = -f_k d = -\mu_k N d = -\mu_k mgd$
- $\Delta K = K_f - K_i = \frac{1}{2}m(v/2)^2 - 0 = \frac{1}{4}(\frac{1}{2}mv^2)$
- $\Delta U = U_f - U_i = 0 - mgh = -mgh$

so $-\mu_k mgd = \frac{1}{4}(\frac{1}{2}mv^2) - mgh = \frac{1}{4}(mgh) - mgh = -\frac{3}{4}mgh$

from first block

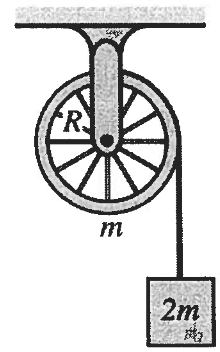
$\mu_k d = \frac{3}{4}h$ but we also know $d = 3h$

$\mu_k(3h) = \frac{3h}{4}$ $\boxed{\mu_k = \frac{1}{4} = 0.25}$

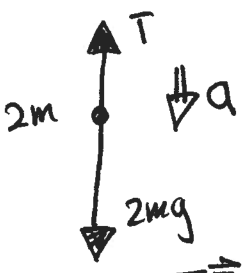
*The following problem will be hand-graded.* <u>*Show all your work for this problem.*</u> *Make no marks and leave no space on your answer card for it.*

**[II]** *(20 points)* A string is wrapped around a pulley wheel of mass $m$ and radius $R$, as shown in the figure. The pulley can rotate freely about its axle, without any friction. The loose end of the string is attached to a block of mass $2m$. You may assume that the spokes of the pulley wheel have negligible mass, so that it may be treated an ideal hoop, with $I_{cm} = mR^2$.

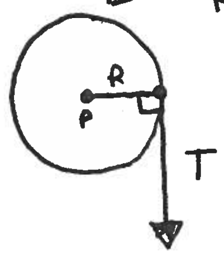
Find an expression for the tension in the cord when the block is released. Express your answer as a multiple of $mg$.




Free Body Diagrams:




$\alpha = \frac{a}{R}$ (no slipping)




$$\Sigma \vec{F}_y = m\vec{a}_y$$

$$\langle +T \rangle + \langle -2mg \rangle = 2m\langle -a \rangle$$

$$\Sigma \vec{\tau}_P = I_P \vec{\alpha}$$

$$\langle -RT \rangle = (mR^2)(-\alpha)$$

$$RT = mR^2\left(\frac{a}{R}\right) = mRa$$

$$a = T/m$$

$$T - 2mg = 2m(-T/m) = -2T$$

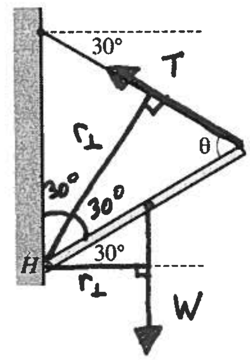
$$3T = 2mg$$

$$\boxed{T = \frac{2}{3}mg}$$

*The following problem will be hand-graded.* *<u>Show all your work for this problem</u>. Make no marks and leave no space on your answer card for it.*

**[III]** *(20 points)* A hinged beam of weight $W$ is attached to a vertical wall, supported in <u>equilibrium</u> by a cable that is attached to the wall above the hinge point. The beam is oriented at a 30° angle *above* the horizontal, while the cable is directed at an angle of 30° *below* the horizontal. Determine the Tension $T$ in the cord, as well as the magnitude and direction of the force $\vec{H}$ exerted by the hinge on the beam.

*Express both force magnitudes as multiples of the beam's weight W. Express the direction of $\vec{H}$ as an angle $\phi$ relative to the horizontal*




Choose pivot at hinge — only $T$, $W$ generate torque about H

- Tension acts ccw, with $r_\perp = L\cos 30^\circ$
- Weight acts cw, with $r_\perp = \frac{L}{2}\cos 30^\circ$

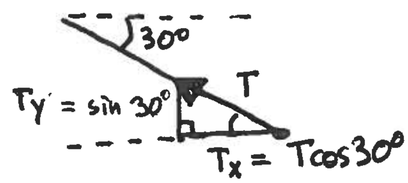
So $\sum \tau_H = 0 = \langle +L\cancel{\cos 30^\circ}\, T\rangle + \langle -\frac{L}{2}\cancel{\cos 30^\circ}\, W\rangle \rightarrow \boxed{T = \frac{W}{2}}$

Now, require $\sum \vec{F}_x = 0$ and $\sum \vec{F}_y = 0$

$\rightarrow$ First, though — express $\vec{T}$ in cartesian form

$\vec{T} = \langle -T\cos 30^\circ\rangle \hat{i} + \langle +T\sin 30^\circ\rangle \hat{j}$

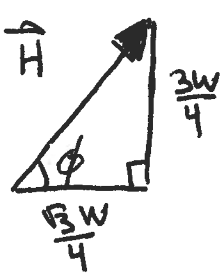
$= \langle -\frac{W}{2}\cdot\frac{\sqrt{3}}{2}\rangle \hat{i} + \langle +\frac{W}{2}\cdot\frac{1}{2}\rangle \hat{j}$




$\boxed{\vec{T} = \langle -\frac{\sqrt{3}}{4}W\rangle \hat{i} + \langle +\frac{W}{4}\rangle \hat{j}}$

Then $\sum \vec{F}_x = \langle +H_x\rangle + \langle -T_x\rangle \longrightarrow \boxed{H_x = T_x = \frac{\sqrt{3}}{4}W}$

$\sum \vec{F}_y = \langle +H_y\rangle + \langle +T_y\rangle + \langle -W\rangle \rightarrow H_y = W - T_y = W - \frac{W}{4} \rightarrow \boxed{H_y = \frac{3W}{4}}$




$|\vec{H}| = \sqrt{\left(\frac{3W}{4}\right)^2 + \left(\frac{\sqrt{3}W}{4}\right)^2} = W\sqrt{\frac{9}{16} + \frac{3}{16}} = W\sqrt{\frac{12}{16}} = W\sqrt{\frac{3}{4}}$

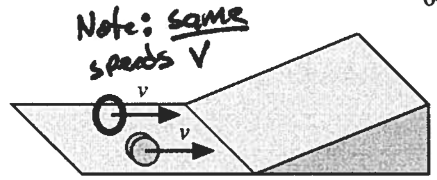
$\boxed{|\vec{H}| = \frac{\sqrt{3}}{2}W}$

directed at angle

$\phi = \tan^{-1}\left(\frac{H_y}{H_x}\right) = \tan^{-1}\left(\frac{\frac{3W}{4}}{\frac{\sqrt{3}W}{4}}\right) = \tan^{-1}(\sqrt{3})$

$\boxed{\phi = 60^\circ}$ above horizontal

Note: same speeds V




*Question value 5 points*

(1) A hoop ($I_{cm} = mr^2$) and a uniform disk ($I_{cm} = \frac{1}{2}MR^2$) are both rolling without slipping along a horizontal surface, when they encounter an upward ramp. Which of the two objects will roll to the highest point on the ramp, before stopping?

(c) The object with the greater radius will roll the highest.
(b) The disk will roll the highest, regardless of mass or radius.
(c) The object with the greater mass will roll the highest.
(d) The hoop will roll highest, regardless of mass or radius.
(e) They will roll to the same height, regardless of mass or radius.

Conservation of Energy:

$K_i + \cancel{U_i}_{\to 0} = \cancel{K_f}_{\to 0} + U_f = mgH$

→ object with greatest rolling KE rises the highest

$K_{hoop} = \frac{1}{2}mv^2 + \frac{1}{2}I_{hoop}\omega^2$
$= \frac{1}{2}mv^2 + \frac{1}{2}(mr^2)(\frac{v}{r})^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$ → $\boxed{\text{Hoop has more rolling KE (per unit mass) and therefore rises higher than disk}}$

$K_{disk} = \frac{1}{2}MV^2 + \frac{1}{2}(\frac{1}{2}MR^2)(\frac{V}{R})^2 = \frac{1}{2}MV^2 + \frac{1}{4}MV^2 = \frac{3}{4}MV^2$

*Question value 5 points*

(2) A conservative force, acting alone, does *negative* work on an object. As a result of this work,

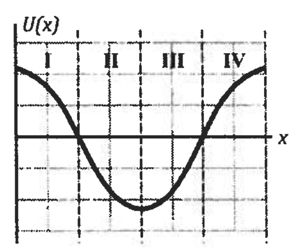
(a) …the potential energy *decreases* and the total mechanical energy *increases*.
(b) …the potential energy *increases* and the total mechanical energy *decreases*.
(c) …the potential energy *increases* and the total mechanical energy *does not change*.
(d) …the potential energy *does not change* and the total mechanical energy *decreases*.
(e) …the potential energy *decreases* and the total mechanical energy *does not change*.

① PE defined by: $\Delta U = -W_{cons}$ → neg $W_{cons}$ means pos $\Delta U$ ⇒ $\boxed{U \text{ increases}}$

② Work by conservative force is an exchange between K and U: $W_{cons}$ means $K \rightleftarrows U$ but total mechanical energy, $\boxed{E = K + U, \text{ will not change}}$

*Question value 5 points*

(3) The graphs at right displays the potential energy curve for a conservative force that acts on a particle that is otherwise free to move along the $x$-axis. In which of the regions shown is the particle experiencing a force in the negative $x$-direction?




(a) In regions II and III.
(b) In regions I and IV.
(c) In regions III and IV.
(d) In none of the regions shown.
(e) In regions I and II.

Force is found as the "negative gradient" of U

⇒ $\vec{F}_x = -\frac{dU}{dx} = -(\text{slope of PE curve})$

So − Force in −x direction would occur when $\boxed{U \text{ has positive slope}}$

⇒ $\boxed{\text{Regions III and IV}}$

*The next two questions involve the following situation:*
A non-uniform rod of mass $M$ and length $L$ lies along the $x$-axis, extending from $x = 0$ to $x = L$. The rod is characterized by a linear mass density that satisfies the expression:

$$\lambda(x) = \frac{2M}{3L}\left(1 + \frac{x}{L}\right) \quad \text{for} \quad 0 \leq x \leq L$$

*Question value 5 points*

(4) What is the $x$-coordinate of the rod's center of mass?

(a) $\cancel{x_{cm} = \frac{1}{2}L}$

(b) $x_{cm} = \frac{2}{3}L$

(c) $\cancel{x_{cm} = \frac{4}{9}L}$

(d) $x_{cm} = \frac{3}{5}L$

(e) $\boxed{x_{cm} = \frac{5}{9}L}$

Note: density is larger at $x=L$ than at $x=0$

→ more mass is concentrated on the right side of the mid point

→ $\boxed{\text{CM should be at some } x > \frac{L}{2}}$

recall that "$\lambda$" represents mass-per-unit-length, so a small mass bit is $dm = \lambda(x)dx$

$$\rightarrow X_{cm} = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_{x=0}^{x=L} x\left[\lambda(x)dx\right] = \frac{1}{M}\int_0^L x \cdot \frac{2M}{3L}\left(1+\frac{x}{L}\right)dx$$

$$= \frac{2}{3L}\int_0^L \left(x+\frac{x^2}{L}\right)dx = \frac{2}{3L}\left[\frac{x^2}{2}+\frac{x^3}{3L}\right]_0^L = \frac{2}{3L}\left[\frac{3L^2}{6}+\frac{2L^3}{6L}\right]$$

$$\boxed{X_{cm} = \frac{5}{9}L}$$

*Question value 5 points*

(5) Lamar, Chloe, and Ralph each have identical copies of the rod described above. Lamar sets a perpendicular pivot axis through the rod at the left end ($x = 0$). Chloe sets a perpendicular pivot axis through the center of mass. Ralph sets a perpendicular pivot axis through the right end of the rod. Starting from rest, all three rods are then spun up to the same final angular speed, $\omega$. Rank, from greatest to least the work that is required to cause each of there rotations.

(a) $\boxed{W_L > W_R > W_C}$

(b) $W_C > W_R > \cancel{W_L}$

(c) $W_L = W_R = W_C$

(d) $W_R > W_L > W_C$

(e) $W_C > W_L = \cancel{W_R}$

① Work–KE theorem, applied to rotation

$$W = \Delta K = K_f - K_i^{\to 0}$$

→ work is greatest in case where final KE is greatest
" " least " " " " " " least

② For all three, $K = \frac{1}{2}I_{pivot}\omega^2$ → greatest moment means greatest KE, and hence, greatest work required

③ Parallel axis theorem $I_p = I_{cm} + Md^2$ ($d$ → distance from P to CM)

(a) Since $I_{cm}$ is the smallest possible moment, $\boxed{W_{cm} \text{ is least}}$

(b) Since CM is closer to right than left, $d_R < d_L \Rightarrow I_R < I_L$

so $W_R < W_L$

Hence, $\boxed{W_L > W_R > W_{CM}}$

*The next two question involve the following situation:*
Object A has mass $m$ and object B has mass $2m$. Starting from rest, both objects are pushed in a straight line along a frictionless surface by force of magnitude $F$.

*Question value 5 points*

(6) Suppose that both objects are pushed through the same total distance $D$ by the force. Compare the work done on objects A and B by force $F$, and also compare the (magnitude of) impulse delivered to objects A and B by force $F$.

(a) $W_B > W_A$ and $J_B = J_A$
(b) $W_B < W_A$ and $J_B > J_A$
(c) $W_B = W_A$ and $J_B = J_A$
(d) $W_B > W_A$ and $J_B < J_A$
(e) $W_B = W_A$ and $J_B > J_A$

① Work is $W = \vec{F} \cdot \Delta\vec{r} = FD$
same force, same distance $\rightarrow$ $\boxed{\text{same Work}}$

② Impulse is $\vec{J} = \vec{F}\Delta t$
since masses are unequal, objects do NOT take the same time to travel distance D

③ since $m_B = 2m_A$, B takes longer to go distance D (accel, B < accel, A)
$\rightarrow \Delta t_B > \Delta t_A \rightarrow \boxed{|\vec{J}_B| > |\vec{J}_A|}$

*Question value 5 points*

(7) Suppose instead that both objects are pushed for the same total time $\Delta t$ by the force. Compare the work done on objects A and B by force $F$, and also compare the (magnitude of) impulse delivered to objects A and B by force $F$.

(a) $W_B = W_A$ and $J_B = J_A$
(b) $W_B > W_A$ and $J_B > J_A$
(c) $W_B = W_A$ and $J_B < J_A$
(d) $W_B < W_A$ and $J_B = J_A$
(e) $W_B < W_A$ and $J_B > J_A$

① Impulse is $\vec{J} = \vec{F}\Delta t$
same force, same time $\rightarrow$ $\boxed{\text{same Impulse}}$

② Work is $W = \vec{F} \cdot \Delta\vec{r} = FD$
but $m_A = \frac{1}{2}m_B \rightarrow a_A = 2a_B$
$\rightarrow$ A will cover twice as much distance in time $\Delta t$
$D_A > D_B$ so $\boxed{W_A > W_B}$

*Question value 5 points*

(8) A baseball is thrown straight upward with a large initial speed $v$. The instantaneous power delivery to the baseball by gravity, as the ball rises to its highest point, is:

(a) constant and *positive* throughout the ball's rise.
(b) *negative* throughout the ball's rise, intially very large but gradually dropping to zero.
(c) initially *zero*, but gradually increaing to a large *negative* value as the ball rises.
(d) always *zero*, because no power is ever delivered by a conservative force like gravity.
(e) constant and *negative* throughout the ball's rise

$P_{inst} = \vec{F} \cdot \vec{V}$ $\rightarrow$ since $\vec{F}$ = downward and $\vec{V}$ = upward, $\boxed{P = \text{negative}}$
$P = -|\vec{F}||\vec{V}| = -mgV$

also, since speed $\downarrow$ as ball rises, $\boxed{|P| \text{ decreases toward zero}}$