Physics 2211 B/C
Spring 2016
Exam 4

Test form 004

Name (printed) Solutions

Recitation Section (see back of test):________

Honor Code Statement:

The work on this test reflects my individual problem-solving efforts, and has not been influenced by any information improperly obtained from another Georgia Tech student during the exam.

*(signed)*________________________________

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number (**ABOVE**) in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

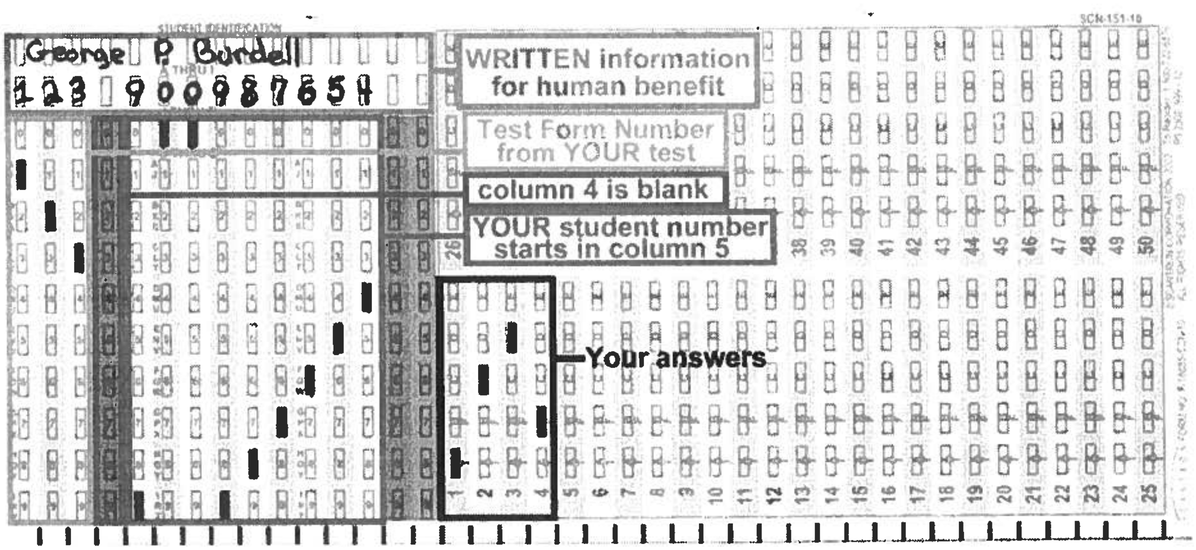
3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

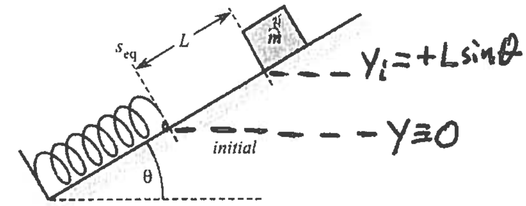
Your test form is: 004



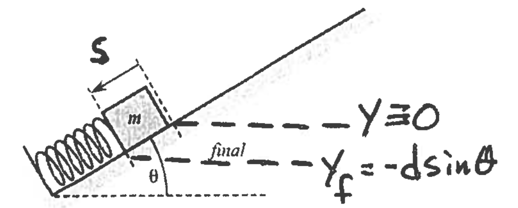
Our final exam will be on Wednesday, May 4
From 6:00 PM to 8:50 PM

The following problem will be hand-graded. <u>*Show all your work for this problem.*</u> *Make no marks and leave no space on your answer card for it.*

[I] *(20 points)* A spring is placed on a frictionless ramp that is inclined 30° above the horizontal. When a block of mass m is placed a distance L above the equilibrium position of the spring (top) and released, the block compresses the spring by a distance $d = L/2$ before stopping (bottom). If the block is <u>instead</u> released a distance $2L$ above the equilibrium position of the spring, how far s will the block compress the spring before stopping? Express s as a fraction of the original distance L.



[Hint: start by finding an expression for the spring constant k in terms of the parameters g, m, and L.]



- Frictionless ramp: $E = K + U = \text{constant}$
- Begins and ends at rest: $K_i = K_f = 0$

$$U_{gi} + \cancel{U_{si}}^{\,0} = U_{gf} + U_{sf}$$

With chosen origin for vertical position,

$$U_{gi} = mgY_i = +mgL\sin\theta$$
$$U_{gf} = mgY_f = -mgd\sin\theta$$

$$+mgL\sin\theta = -mgd\sin\theta + \tfrac{1}{2}kd^2$$
$$\rightarrow \tfrac{1}{2}kd^2 = mg(L+d)\sin\theta$$

Now, use $d = L/2$: $\frac{1}{2}k\frac{L^2}{4} = mg\left(\frac{3}{2}L\right)\sin\theta$

$$k = \frac{12mg}{L}\sin\theta = \frac{12mg}{L}\cdot\frac{1}{2} = \boxed{\frac{6mg}{L} = k}$$

Now — same problem, but let $Y_i = 2L\sin\theta$, $Y_f = -s\sin\theta$, <u>s = unknown</u>

$$U_{gi} + \cancel{U_{si}}^{\,0} = U_{gf} + U_{sf}$$

$$+mg(2L)\sin\theta = -mgs\cdot\sin\theta + \tfrac{1}{2}ks^2$$

$\sin\theta = \sin 30° = 1/2$, $k = \frac{6mg}{L}$

$$\cancel{mg}L = -\frac{\cancel{mg}}{2}s + \frac{1}{2}\left(\frac{6\cancel{mg}}{L}\right)s^2$$

$\rightarrow$ multiply through by $2L$:

$$2L^2 = -Ls + 6s^2 \rightarrow 6s^2 - Ls - 2L^2 = 0$$

$$s = \frac{-(-L) \pm \sqrt{(-1)^2 - 4(6)(-2)L^2}}{2(6)}$$

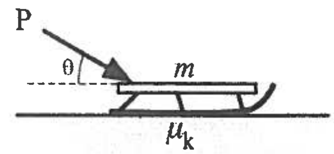
$$= \frac{L \pm \sqrt{49L^2}}{12} = \frac{L \pm 7L}{12}$$

by definition, s <u>must</u> be positively valued, so choose <u>pos</u> root

$$s = \frac{8L}{12} \Rightarrow \boxed{s = \tfrac{2}{3}L}$$

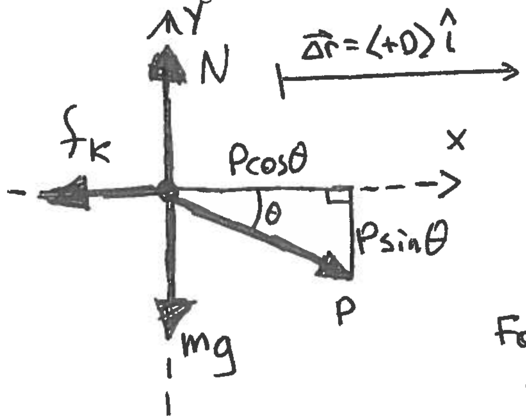
The following problem will be hand-graded. Show all your work for this problem. Make no marks and leave no space on your answer card for it.

[II] *(20 points)* An empty sled of mass m is at rest on a level patch of rough snow having coefficient of friction $\mu_k = 0.25$. A child begins to push the sled, using an applied force $P = 2mg$ directed at an angle $\theta = 30°$ below the horizontal. He pushes a total distance D in a staight line before letting go (and then hopping onto the sled, but that's not part of the problem...).



Draw a free body diagram for the sled while it is being pushed. ***The quality of your diagram will be a graded part of this question!*** Write out expressions for the work done by every force that acts on the sled. In each case, express the work in terms of D, g, m, μ_k, and θ. (If a particular force does no work, be sure to explain why it does so.)

Now, let $D = 1.5$ meters, and use the Work-Kinetic Energy Theorem to determine the final speed of the sled at the moment the child lets go.



Normal force does zero work: $\vec{N} \perp \vec{\Delta r}$
(vertical) ⊥ (horizontal)

Ditto for gravitational force

$\boxed{W_N = 0}$ $\boxed{W_{grav} = 0}$

For pushing force P:

$$W_P = \vec{P} \cdot \vec{\Delta r} = P_x \Delta X = (P\cos\theta)(+D)$$

$$\boxed{W_P = +PD\cos\theta = +2mgD\underbrace{\cos\theta}_{\sqrt{3}/2}} = \sqrt{3}mgD$$

Finally, work done by friction

$f_k = \mu_k N$

→ must apply 2nd Law: $\sum \vec{F}_y = 0 = \langle +N \rangle + \langle -mg \rangle + \langle -P\sin\theta \rangle$

$$N = mg + (2mg)\sin\theta = mg(1 + 2\sin\theta) = 2mg$$

so: $f_k = \mu_k mg(1 + 2\sin\theta)$

Since $\vec{f_k}$ is opposite to $\vec{\Delta r}$, dot product gives factor (−1)

$$\boxed{W_f = -\underbrace{\mu_k}_{0.25} mgD\underbrace{(1 + 2\sin\theta)}_{0.5}} = -0.5mgD$$

$$W_{TOT} = \Delta K$$

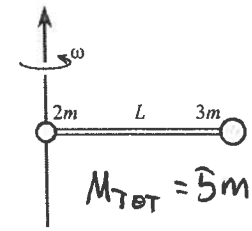
$$\underbrace{\cancel{W_N}}_{0} + \underbrace{\cancel{W_g}}_{0} + W_P + W_f = K_f - \underbrace{\cancel{K_i}}_{0}$$

$$(\sqrt{3}mgD) + \left(-\tfrac{1}{2}mgD\right) = \tfrac{1}{2}mV_f^2$$

$$\boxed{V_f = \sqrt{(2\sqrt{3} - 1)gD} = 6.0 \text{ m/s}}$$

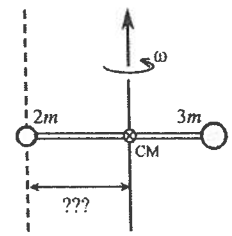
The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[III] *(20 points)* Two point masses ($2m$ and $3m$) are attached via a rigid, massless rod of length L, as shown at top right. A massless vertical axle is passed through the left end of the assembly (i.e. directly through mass $2m$) as shown, and the whole assembly is given an angular speed ω. The work required to cause this rotation, beginning from rest, is W_{left}. (Note: the Work-Kinetic Energy Theorem applies to rotations just as it applies to translations…)



$M_{TOT} = 5m$

Suppose instead that the vertical axle is passed through the center of mass of the assembly, as shown in the bottom figure. Determine the amount of work W_{CM} that would be required to give the new assemply the same rotational speed ω (again starting from rest). Express your answer as a multiple (or fraction) of W_{left}.



You will need to start by locating the assembly's center of mass!

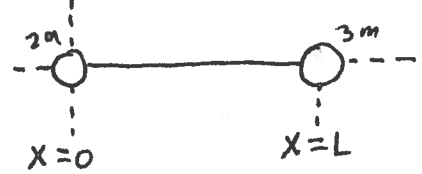
$$W = \Delta K_{rot} = K_f - \cancel{K_i}^{\,0} = \frac{1}{2} I_{pivot}\,\omega^2$$

for pivot at left side:

$$I_{left} = \sum_i m_i r_i^2 = (2m)(0)^2 + (3m)L^2 = 3mL^2$$

$$\rightarrow W_{left} = \frac{3}{2} mL^2\omega^2$$

Now, locate actual CM



$$X_{cm} = \frac{1}{M_{TOT}} \sum_i m_i x_i = \frac{1}{2m+3m}\left[2m\cdot 0 + 3m\cdot L\right]$$

$$X_{cm} = \frac{3mL}{5m} \qquad \boxed{X_{cm} = \frac{3}{5}L}$$

So, with pivot at CM, $I_{cm} = \underbrace{2m\left(\frac{3}{5}L\right)^2}_{\text{distance from 2m to CM}} + \underbrace{3m\left(\frac{2}{5}L\right)^2}_{\text{distance from 3m to CM}}$

$$I_{cm} = \frac{18}{25}mL^2 + \frac{12}{25}mL^2$$

$$I_{cm} = \frac{30}{25}mL^2 = \boxed{\frac{6}{5}mL^2}$$

Then $W_{cm} = \Delta K_{rot,cm} = K_f - K_i = \frac{1}{2} I_{cm}\,\omega^2 - 0$

$$\Rightarrow \frac{W_{cm}}{W_{left}} = \frac{\frac{1}{2}I_{cm}\omega^2}{\frac{1}{2}I_{left}\omega^2} = \frac{I_{cm}}{I_{left}} = \frac{\frac{6}{5}mL^2}{3mL^2} = \frac{2}{5}$$

$$\boxed{W_{cm} = \frac{2}{5}W_{left} = 0.4\,W_{left}}$$

one could also use parallel axis theorem

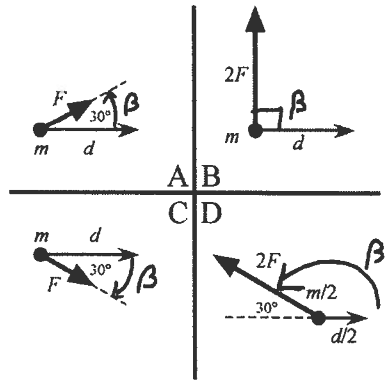
$$I_{left} = I_{cm} + \underbrace{(5m)}_{M_{TOT}}\underbrace{\left(\frac{3}{5}L\right)^2}_{\text{distance from CM to axis at left}}$$

then

$$I_{cm} = I_{left} - \frac{9}{5}ML^2 = \frac{15}{5}ML^2 - \frac{9}{5}ML^2 = \frac{6}{5}ML^2$$

Question value 5 points

(1) In the figures at right, four ojects (labelled by their masses) are acted upon by the indicated force vectors (labelled by their *magnitudes*), as the objects experience the indicated vector displacements (also labelled by *magnitudes*). Rank, from most positive to most negative, the work done on obects A–D by theses forces.



(a) $W_A > W_C > W_B > W_D$

(b) $\boxed{W_A = W_C > W_B > W_D}$

(c) $W_D > W_C = W_A > W_B$

(d) $W_D > W_B > W_A > W_C$

(e) $W_C > W_A > W_D > W_B$

$W = \vec{F} \cdot \Delta\vec{r}$
$= |\vec{F}|\,|\Delta\vec{r}| \cos\beta$

$\rightarrow W_A = Fd\cos 30^\circ$

$\rightarrow W_B = 2Fd\cancel{\cos 90^\circ} = 0$

$\rightarrow W_C = Fd\cos 30^\circ$ [angle being below horizontal is irrelevant]

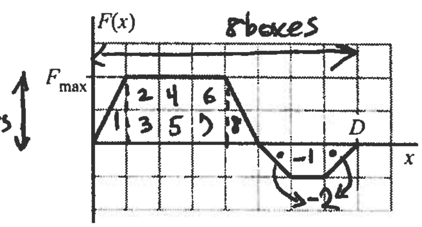
$\rightarrow W_D = (2F)\left(\frac{d}{2}\right)\underbrace{\cos(180-30)}_{=-\cos 30^\circ}$

$= -Fd\cos 30^\circ$

$W_A = W_C$
$W_B = 0$
$W_D = -W_A = -W_C$

Question value 5 points

(2) A particle moves along the x-axis, subject only to a varying x-directed force, graphed at right as a function of position. How much work does the force do on the particle, as the particle moves between $\vec{x} = 0$ and $\vec{x} = \langle +D \rangle$?



(a) $\frac{1}{2}F_{max}D$

(b) $\frac{3}{4}F_{max}D$

(c) $\frac{5}{8}F_{max}D$

(d) $\frac{3}{5}F_{max}D$

(e) $\boxed{\frac{3}{8}F_{max}D}$

When $\vec{F}$ is a variable force,

$W = \int \vec{F}(x) \cdot d\vec{x}$ = area under curve

From graph: area above = +8 boxes, area below = −2 boxes] total area = $\boxed{+6 \text{ boxes}}$

Now: height of one box is $\frac{F_{max}}{2}$, width of one box is $\frac{D}{8}$] one box equals $\frac{F_{max}D}{16}$ of work] $\rightarrow$ Total work is $6 \cdot \frac{F_{max}D}{16} = \boxed{\frac{3}{8}F_{max}D}$

Question value 5 points

(3) A cable is used to winch a pallet of roofing tiles from ground level to the top of a five-story building. During this process, the only forces acting on the pallet are tension and gravity; there is no significant aerodynamic drag to consider. Consider the pallet while it is rising at constant speed, partway through the process. Describe the work done by tension, W_T, the work done by gravity, W_g, and the energy changes they cause.

(a) W_T is zero leaving the kinetic energy unchanged, while W_g is positive and adds to the potential energy.

(b) W_T is positive and adds to the mechanical energy, while W_g is negative and adds to the potential energy.

(c) W_T is positive and adds to the potential energy, while W_g is zero and leaves the mechanical energy unchanged.

(d) W_T is negative and subtracts from the mechanical energy, while W_g is positive and adds to the potential energy.

(e) W_T is and W_g are both zero, leaving the mechanical and potential energies unchanged..

Constant speed — total work is zero

W_T adds energy to system, from outside: Total Mech Energy increases $E_{TOT}\uparrow$ (pos work)

W_g transfers energy from kinetic to Potential: $U\uparrow$, but E_{TOT} unchanged by gravity (neg work)

Question value 5 points

(4) An object moving to the right is acted upon by a constant force such that the object experiences a leftward acceleration. What can be said about the power delivered to the object by the force?

(a) The power is positively-valued, and remains constant as time passes.
(b) The power is negatively-valued, and remains constant as time passes.
(c) The power is positively-valued, and increases in magnitude as time passes.
(d) The power is negatively-valued, and increases in magnitude as time passes.
$\boxed{\text{(e) The power is negatively-valued, and decreases in magnitude as time passes.}}$

let right = positive

- $\vec{V} = \langle +V_o \rangle$ — magnitude, direction
- $\vec{a} = \langle -a_o \rangle$

so $\vec{F} = \langle -ma_o \rangle$

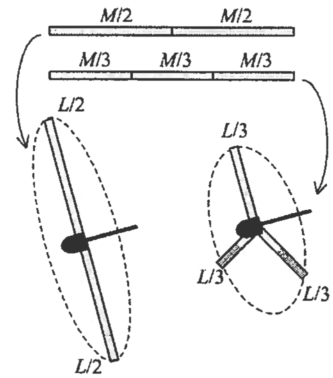
- $\vec{a}$ opposite to $\vec{V}$: $|\vec{V}|$ decreasing

Power: $P = \vec{F}\cdot\vec{V} = -m a_o V_o$

$\boxed{\text{negative value decreasing in magnitude}}$

Question value 5 points

(5) A thin rod of mass M and length L, when rotated about an axis through its endpoint, has a moment of inertia $I_{\text{end}} = ML^2/3$. Starting with two identical rods, you create a 2-blade propellor by breaking one rod into two equal parts, and attaching their ends to a hub, and a 3-blade propellor by breaking the rod into three equal parts and attaching their ends to another hub. (Assume both propellor hubs have negligible mass.) Compare the moment of inertia of the 3-blade prop to the moment of inertia of the 2-blade prop.



$\boxed{(a)\ I_3 = \frac{4}{9} I_2}$
(b) ~~$I_3 = \frac{3}{2} I_2$~~
(c) ~~$I_3 = \frac{9}{4} I_2$~~
(d) ~~$I_3 = I_2$~~
(e) $I_3 = \frac{2}{3} I_2$

Two-blade:
$$I_2 = 2\times\left[\frac{m}{2}\frac{(L/2)^2}{3}\right] = \frac{mL^2}{12}$$

Three-blade:
$$I_3 = 3\times\left[\frac{m}{3}\frac{(L/3)^2}{3}\right] = \frac{mL^2}{27}$$

$$\frac{I}{I} = \frac{mL^2}{mL^2} = \frac{12}{27} = \frac{4\cdot 3}{9\cdot 3} = \frac{4}{9}$$

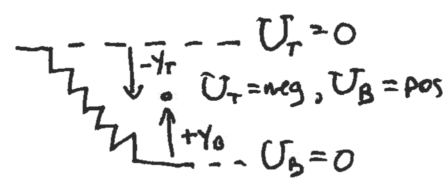
$$\boxed{I_3 = \frac{4}{9} I_2}$$

Since 3-blade has more of its mass near the hub, we would naturally expect $I_3 < I_2$

Question value 5 points

(6) Tom stands at the top of a stairwell, and defines $U_T = 0$ at his location. Ben stands at the bottom the same stairwell, and defines $U_B = 0$ at his location. Tom tosses a soda can down to Bob. While it is in flight, what do Tom and Ben infer about the kinds of energy possessed by the can?

(a) Ben sees less kinetic energy and less total mechanical energy.
(b) Tom sees less kinetic energy, but more total mechanical energy.
$\boxed{\text{(c) They both see the same kinetic energy, but Ben sees more total mechanical energy.}}$
(d) They both see the same kinetic energy, but Tom sees more total mechanical energy.
(e) They both see the same kinetic energy and the same total mechanical energy



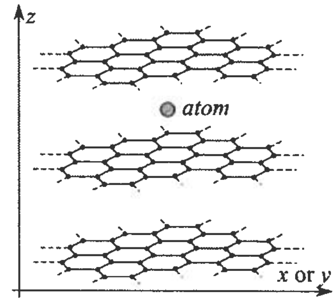
They are both at rest, so they see the same velocity hence same speed → hence $\boxed{\text{Same Kinetic Energy}}$

$E_{TOT} = K + U$ → since ball is always above ~~Ben~~Bob, $U_B = \text{pos}$
" " " " below Tom, $U_T = \text{neg}$

→ $U_B > U_T$

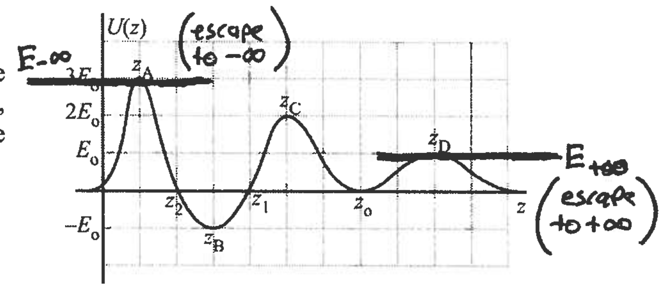
$$\boxed{E_B > E_T}$$

The next two questions involve the following situation:
The picture at top right shows a schematic side view of a neutral atom trapped within a three-layer graphene film. (Graphene is simply a flat, hexagonally shaped sheet of carbon atoms.) Below that, the total potential energy of the atom is plotted as a function of the vertical position z within the tri-layer film. No external or dissipative forces act. The atom begins at position z_0.



Question value 5 points

(7) In which regions listed below would the atom experience a force in the *downward* (i.e. negative-z) direction? (If a range of positions is listed, the atom must experience a downward force at all positions within the given range.)



(a) Everywhere beyond position z_D.
(b) Between positions z_2 and z_1.
$\boxed{\text{(c) Between positions } z_B \text{ and } z_C.}$
(d) Between positions z_A and z_B.
(e) Only exactly at points z_A, z_C, and z_D.

Force = "gradient" of PE function ("slope")

$\vec{F}_z = \langle -\frac{dU}{dz} \rangle$

Force in negative direction: $\frac{dU}{dz}$ = positive
slope = positive

→ true between $z=0$ and $z=z_A$, between z_0 and z_D, and $\boxed{\text{between } z_B \text{ and } z_C}$

Question value 5 points

(8) What minimum kinetic energy must the atom have at z_0 to escape from the film? [Assume the film has infinite extent in the horizontal x- and y-directions, so no motion in those directions allow a true escape.]

$\boxed{\text{(a) The minimum kinetic energy for the atom to escape is } K = E_0}$
(b) There is no kinetic energy value for which the atom can escape.
(c) The minimum kinetic energy for the atom to escape is $K = 3E_o$.
Absurd! ~~(d) The minimum kinetic energy for the atom to escape is $K = -E_0$.~~
(e) The minimum kinetic energy for the atom to escape is $K = 2E_o$.

"Escape" simply means being free to travel to either $z=+\infty$ or $z=-\infty$

- To get to $-\infty$, atom must have enough total energy to get over:
  hill at z_C → total energy $2E_0$
  hill at z_A → total energy $3E_0$] minimum energy to escape to $-\infty$

  But: Particle starts at z_0, where $U=0$
  If total energy must be $K+U=3E_0$, atom would require $K=+3E_0$

- to get to $+\infty$, atom only needs enough total energy to get over hill at z_D, requiring total energy E_0
  Since $U=0$ at z_0, atom must have $\boxed{K=+E_0}$ → min KE to escape is thus $+E_0$, and escape is to $+\infty$