I. (16 points) A block of mass m is sliding up a vertical wall at constant non-zero velocity $\vec{v}_0$, due to an applied force $\vec{P}$ pushing against it at an angle θ above the horizontal, as shown. The coefficient of static friction between the block and the wall is μ_s, and the coefficient of kinetic friction is μ_k. What is the magnitude of the applied force? Express this magnitude of $\vec{P}$ in terms of other parameters defined in the problem, and physical or mathematical constants. (*On Earth.*)
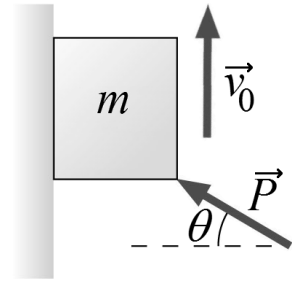
Use Newton's Second Law. Sketch a Free Body Diagram. There will be a gravitational force $m\vec{g}$ downward, a normal force $\vec{n}$ to the right, a force of kinetic friction $\vec{f}_k$ downward (as the block is sliding upward), and the push force $\vec{P}$ up and to the left at the angle θ. Choose a coordinate system. I've chosen y upward and x to the left.
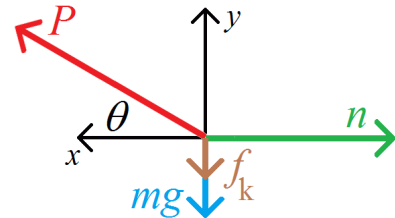
Write out Newton's Second Law (the sum of the force is ...) for each axis. I'll show signs explicitly, so symbols represent magnitudes. First the x direction,

$$\sum F_x = P_x - n = ma_x = 0 \qquad \Rightarrow \qquad P\cos\theta = n$$

then the y direction

$$\sum F_y = P_y - f_k - mg = ma_y = 0 \qquad \Rightarrow \qquad P\sin\theta = \mu_k n + mg$$

Substituting the expression for n found from the x dimension

$$P\sin\theta = \mu_k P\cos\theta + mg$$

Solve for P.

$$P\sin\theta - \mu_k P\cos\theta = mg \qquad \Rightarrow \qquad P(\sin\theta - \mu_k\cos\theta) = mg \qquad \Rightarrow \qquad P = \frac{mg}{\sin\theta - \mu_k\cos\theta}$$

II. (16 points) Mars has a radius of $3397\,\text{km}$ and rotates once on its axis every 1.03 days (or $88{,}990$ seconds). What is the centripetal acceleration magnitude, in m/s^2, of an object on Mars' surface, located as shown at $65°$ latitude? Sketch the direction of the object's centripetal acceleration on the diagram.
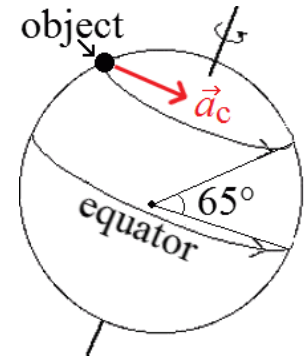
The centripetal acceleration must point directly toward the center of the circular path followed by the object.

The magnitude of the centripetal acceleration can be related to the object's tangential speed v, and to Mar's period T, through Mars' angular speed ω.

$$a_c = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2 = r\left(\frac{2\pi}{T}\right)^2$$

Note that the radius of the circular path r, is not the same as Mars' radius R. The farther from the equator the object is, the smaller the radius of its path.

$$a_c = R\cos\theta\left(\frac{2\pi}{T}\right)^2 = \left(3397\times10^3\,\text{m}\right)\cos 65°\left(\frac{2\pi}{8.899\times10^4\,\text{s}}\right)^2 = 0.0072\,\text{m/s}^2$$

1. (6 points) In the problem above, compare the angular speed and centripetal acceleration magnitude of the object at $65°$ latitude, to those of an object on Mars' equator.

Every point on Mars' requires the same time to go once around its axis, so every point has the same angular speed ω. The centripetal acceleration of each point, however, depends on how far it is from the axis, $a_c = r\omega^2$. An object at $65°$ latitude is closer to the axis, so

The object at $65°$ latitude has **the same** angular speed, but **lesser** centripetal acceleration.

III. (16 points) You are playing soccer and kick the ball over a fence off the pitch. Your friend goes to retrieve the ball and has to kick it over a 10.0 m fence. She can kick a soccer ball at 28.0 m/s. She repositions the ball to minimize the angle above horizontal that she needs to kick to clear the fence. What is that minimum angle? (*On Earth.*)

Once kicked, the ball acts under the influence of only gravity, so the principles of projectile motion can be used to solve this problem. Projectile motion is a special case of constant-acceleration kinematics, with an acceleration of zero horizontally, and an acceleration of g downward vertically. Consider the vertical motion:

$$v_y = v_{0y} + a_y \, \Delta t \qquad \text{and} \qquad y = y_0 + v_{0y} \, \Delta t + \tfrac{1}{2} a_y \, (\Delta t)^2$$

Eliminate Δt. I'll place the origin at the location where the ball is kicked, with the x axis horizontal toward the fence, and y vertically upward. At minimum kick angle θ, the ball will just clear the fence when it reached the peak of its trajectory. Find an expression for the time required for the ball to reach this point.

$$v_y = v_{0y} + a_y \, \Delta t \qquad \Rightarrow \qquad 0 = v_0 \sin \theta - g \, \Delta t \qquad \Rightarrow \qquad \Delta t = \frac{v_0 \sin \theta}{g}$$

Then

$$y = y_0 + v_{0y} \, \Delta t + \tfrac{1}{2} a_y \, (\Delta t)^2 \qquad \Rightarrow \qquad y = 0 + v_0 \sin \theta \left(\frac{v_0 \sin \theta}{g} \right) - \tfrac{1}{2} g \left(\frac{v_0 \sin \theta}{g} \right)^2$$

Solve for θ.

$$2gy = v_0^2 \sin^2 \theta \qquad \Rightarrow \qquad \sin \theta = \frac{\sqrt{2gy}}{v_0}$$

So

$$\theta = \sin^{-1} \left(\frac{\sqrt{2gy}}{v_0} \right) = \sin^{-1} \left(\frac{\sqrt{2 \, (9.81 \, \text{m/s}^2) \, (10.0 \, \text{m})}}{28.0 \, \text{m/s}} \right) = 30.0°$$

2. (6 points) If θ_c is the minimum angle she must kick, is there another angle that she can kick so that the ball travels the same horizontal distance before striking the ground?

Maximum distance is achieved with a kick at 45°, with the distances falling off symmetrically with angles on either side of that. So

$$\text{Yes, } 90° - \theta_c.$$

3. (8 points) The pendulum bob, A, was struck by the bat, and is now swinging upward. How many of the items on this list are forces acting on the bob as it swings upward? (*On Earth.*)

- $m\vec{a}$
- gravitational force
- centripetal force
- normal force
- tension force
- strike force

. . . .

The bob isn't against a surface, so no normal force acts on it. Neither $m\vec{a}$ nor "centripetal force" are forces—instead they are what forces add to. The strike force no longer acts on the bob after it leaves contact with the bat. Only the tension force and gravitational force are acting on the bob as it swings up, so the number of forces is

2
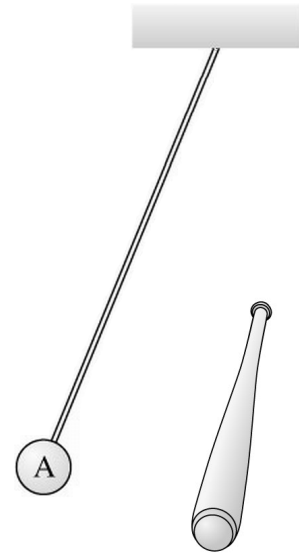
4. (8 points) A person stands on an ordinary bathroom spring scale while in an elevator. In which of these situations, if any, will the scale readings be the same? (*On Earth.*)

  i) The elevator is moving upward at $3\,\mathrm{m/s}$ and the speed is increasing at $2\,\mathrm{m/s^2}$.

  ii) The elevator is moving downward at $3\,\mathrm{m/s}$ and the speed is increasing at $2\,\mathrm{m/s^2}$.

  iii) The elevator is moving downward at $3\,\mathrm{m/s}$ and the speed is decreasing at $2\,\mathrm{m/s^2}$.

. . . .

When the scale readings are the same, the net force on the person will be the same, so the accelerations will be the same.

In i, the acceleration is $2\,\mathrm{m/s^2}$ upward. In ii, the acceleration is $2\,\mathrm{m/s^2}$ downward. In iii, the acceleration is $2\,\mathrm{m/s^2}$ upward. So,

Only situations i and iii.

Spring scale
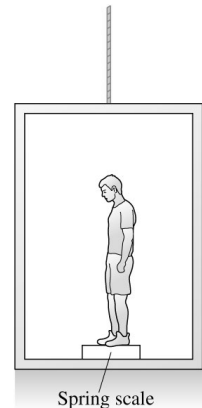
5. (8 points) A mom is trying to get her family home. Her $30\,\mathrm{kg}$ child is throwing a fit and won't move. In an attempt to get home, she pulls him to the left with a force of $\vec{F}_M = 400\,\mathrm{N}$. To make matters worse, the golden retriever the child is holding has seen a squirrel in the opposite direction and is pulling to the right with a force of $\vec{F}_D = 300\,\mathrm{N}$. His shoes have a coefficient of static friction of 0.5. Which free body diagram best represents this situation? (*On Earth.*)

. . . .

The Mom's force $\vec{F}_M$ is to the left, and the dog's force $\vec{F}_D$ is to the right. The gravitational force is downward, and the normal force is upward. Only one of the offered Free Body Diagrams satisfies all these conditions.
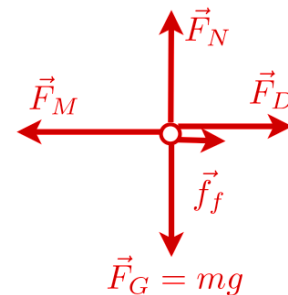
6. (8 points) In the problem above, how much harder, if at all, must his mom pull him to start the family moving?

. . . .

Noting that the normal force must balance the gravitational force on the child, the *maximum* force of static friction is

$$f_{s,\mathrm{max}} = \mu_s n = \mu_s mg = 0.5\,(30\,\mathrm{kg}(\,(9.81\,\mathrm{m/s^2}) = 147\,\mathrm{N}$$

so the maximum rightward force is the sum of the frictional and dog forces, $147\,\mathrm{N} + 300\,\mathrm{N} = 447\,\mathrm{N}$

Since she's already pulling leftward with a force of $400\,\mathrm{N}$, to overcome friction and the dog, she must pull with an additional

$47\,\mathrm{N}$ (for a total force of $447\,\mathrm{N}$)

7. (8 points) I am trying to get across an airport terminal to a gate that is $500\,\mathrm{m}$ away, so I take a moving sidewalk part of the way which moves at a constant speed of $1\,\mathrm{m/s}$. The whole time I walk at a constant speed of $3\,\mathrm{m/s}$. The trip takes me 2.5 minutes (or 150 seconds). How long was the moving sidewalk? (*Hint:* write an expression for the total distance traveled and solve for the time on the moving sidewalk. Don't forget to check your answer!)

. . . .

Let D be the total distance, and d_w be the distance walking, and d_s be the distance on the moving sidewalk. Let T be the total time, and t_w be the time walking, and t_s be the time on the moving sidewalk. Then

$$D = d_w + d_s = v_w t_w + v_s t_s$$

where v_w is my speed while walking on the ground, and v_s is my speed (with respect to the ground) while walking on the moving sidewalk. Use an expression for the total time to eliminate the time walking on the ground.

$$T = t_w + t_s \quad \Rightarrow \quad t_w = T - T_s \quad \text{so} \quad D = v_w\left(T - t_s\right) + v_s t_s$$

Solve for t_s.

$$D = v_w T - v_w t_s + v_s t_s = v_w T + \left(v_s - v_w\right) t_s \quad \Rightarrow \quad t_s = \frac{D - v_w T}{v_s - v_w}$$

The length of the sidewalk is the time on the sidewalk times my speed (with respect to the ground) on the sidewalk. Note that this is the sum of the speed of the sidewalk with respect to the ground and my speed with respect to the sidewalk, so $v_s = 1\,\mathrm{m/s} + 3\,\mathrm{m/s} = 4\,\mathrm{m/s}$.

$$d_s = v_s t_s = v_s\left(\frac{D - v_w T}{v_s - v_w}\right) = (4\,\mathrm{m/s})\left(\frac{500\,\mathrm{m} - (3\,\mathrm{m/s})\,(150\,\mathrm{s})}{4\,\mathrm{m/s} - 3\,\mathrm{m/s}}\right) = 200\,\mathrm{m}$$