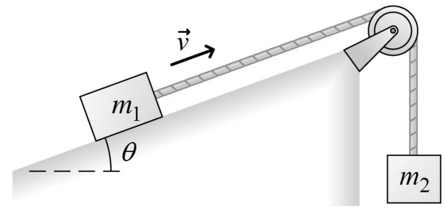
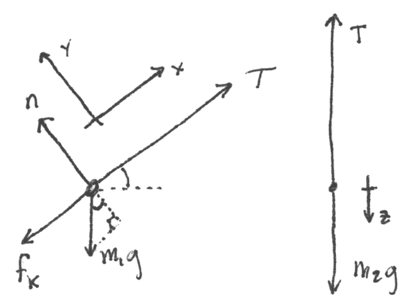
I. (16 points) A block of mass m_1 is connected by an ideal rope passing over an ideal pulley to a block of mass m_2. The block of mass m_1 slides up a plane that makes an angle θ with the horizontal, as shown. If each block has an acceleration magnitude a, what is the coefficient of kinetic friction between m_1 and the plane? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants. (*On Earth.*)



Use Newton's Second Law. Sketch a Free Body Diagram of each block. The block of mass m_1 has a tension T up along the plane, a kinetic friction force f_k down along the plane (since it's sliding up), a normal force n up perpendicular to the plane, and a gravitational force $m_1 g$ downward. The block of mass m_2 has a tension force T upward, and a gravitational force $m_2 g$ downward. Choose a coordinate system for each Free Body Diagram. For the block m_1, I'll choose the positive x axis up along the plane, and the positive y axis perpendicular to the plane. This choice makes $a_x = a$ and $a_y = 0$. For the block m_2, I'll choose positive the positive z axis downward, so $a_z = a$.



Write Newton's Second Law for each axis. I'll show signs explicitly, so symbols represent magnitudes. For block m_2,

$$\sum F_z = m_2 g - T = m_2 a_z = m_2 a \qquad \Rightarrow \qquad T = m_2 g - m_2 a$$

For block m_1,

$$\sum F_y = n - m_1 g \cos\theta = m_a a_y = 0 \qquad \Rightarrow \qquad n = m_1 g \cos\theta$$

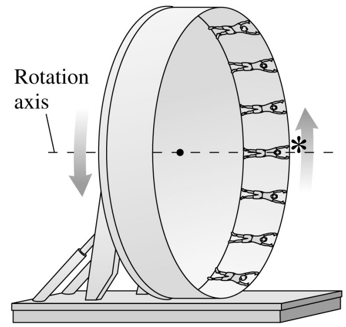
$$\sum F_x = T - m_1 g \sin\theta - f_k = m_1 a_x = m_1 a \quad \Rightarrow \quad T = m_1 a + m_1 g \sin\theta + \mu_k n = m_1 a + m_1 g \sin\theta + \mu_k m_1 g \cos\theta$$

where the force of kinetic friction is $f_k = \mu_k n$. Eliminate the unknown tension by setting the two expressions for it equal to each other, and solve for μ_k.

$$T = m_2 g - m_2 a = m_1 a + m_1 g \sin\theta + \mu_k m_1 g \cos\theta \qquad \Rightarrow \qquad \mu_k m_1 g \cos\theta = m_2 g - m_2 a - m_1 a - m_1 g \sin\theta$$

$$\mu_k = \frac{m_2 g - m_2 a - m_1 a - m_1 g \sin\theta}{m_1 g \cos\theta}$$

II. (16 points) In an amusement park ride called *The Roundup*, passengers stand inside a ring of radius R. After the ring has acquired sufficient angular speed, it tilts into a vertical plane, as shown. If the wall of the ride exerts a normal force $mg/3$ on a passenger of mass m at the topmost position, what normal force is exerted on that same passenger at the bottom position? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants. (*On Earth.*)



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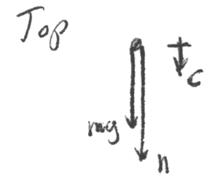
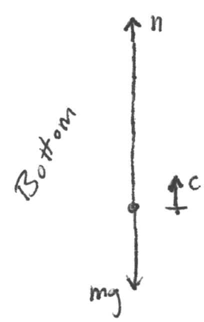
Use Newton's Second Law. Sketch a Free Body Diagram of the passenger at the top. There will be a gravitational force mg downward, and a normal force n downward. Choose an axis that points in the direction of the acceleration, which is down in this case. Write Newton's Second Law for this passenger at the top. I'll show signs explicitly, so symbols represent magnitudes.



$$\sum F_c = n + mg = ma_c \qquad \Rightarrow \qquad ma_c = \frac{mg}{3} + mg = \frac{4}{3}mg$$

Now sketch a Free Body Diagram of the passenger at the bottom. There will be a gravitational force mg downward, and a normal force n upward. Choose an axis that points in the direction of the acceleration, which is up in this case. Write Newton's Second Law for this passenger at the bottom. Note that the magnitude of the acceleration is now known.



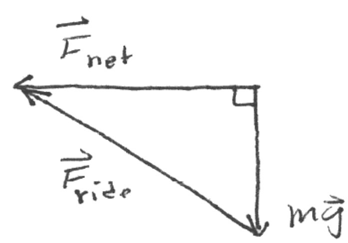
$$\sum F_c = n - mg = ma_c \qquad \Rightarrow \qquad n = mg + ma_c = mg + \frac{4}{3}mg = \frac{7}{3}mg$$

1. (6 points) Once the ride has moved into the vertical plane, what is the direction of the **total** force exerted **by** the ride **on** a passenger who is exactly halfway between the top and bottom, as marked with an asterisk in the figure? (*Hint:* consider the direction of the net force on the passenger.)

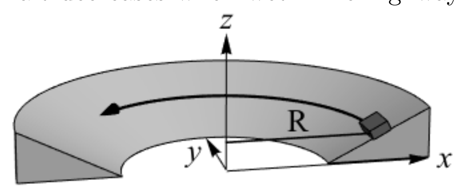
. . . .

Two agents exert force on the passenger, the Earth and the ride. Since the net force on a passenger halfway up must be horizontal (toward the center), and the gravitational force from the Earth is downward, the total force exerted **by** the ride **on** the passenger must have an upward component. The force from the ride must be

Between horizontal and vertically upward.



III. (16 points) There is a turn on the road (of radius R) out to Sweetwater Creek State Park that was designed for traffic traveling at the speed limit, but most people take it faster. When it rains, this causes a lot of accidents because the coefficient of static friction μ_s between rubber and asphalt decreases when wet. The highway engineers want to minimize accidents, so they they change the bank of the curve to an angle θ from the horizontal. What is the maximum speed at which a vehicle could safely make it around the curve with wet asphalt? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants. (*On Earth.*)



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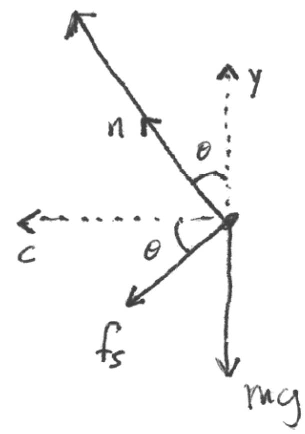
Use Newton's Second Law. Sketch a Free Body Diagram of the car—a "rear view" is most helpful. There will be a gravitational force mg downward, a static frictional force f_s down the slope (as we're concerned with the *maximum* speed of the car—friction prevents it from sliding to the outside of the curve), and a normal force n perpendicular to the road surface. Choose a coordinate system. Since the acceleration of the car must be toward the center of the curve (horizontal!) I'll choose that as the positive c direction. The other axis must be perpendicular, so I'll choose the positive y direction to be straight upward.



Write Newton's Second Law for each axis. I'll show signs explicitly, so symbols represent magnitudes.

$$\sum F_y = n\cos\theta - f_s\sin\theta - mg = ma_y = 0 \quad \Rightarrow \quad n\cos\theta - \mu_s n\sin\theta = mg$$

$$\Rightarrow \quad n = \frac{mg}{\cos\theta - \mu_s\sin\theta}$$

Then

$$\sum F_c = n\sin\theta + f_s\cos\theta = ma_c = m\frac{v^2}{r} \quad \Rightarrow \quad n\sin\theta + \mu_s n\cos\theta = m\frac{v^2}{R} \quad \Rightarrow \quad n\left(\sin\theta + \mu_s\cos\theta\right) = m\frac{v^2}{R}$$

Eliminate the unknown normal force by substitution, and solve for v.

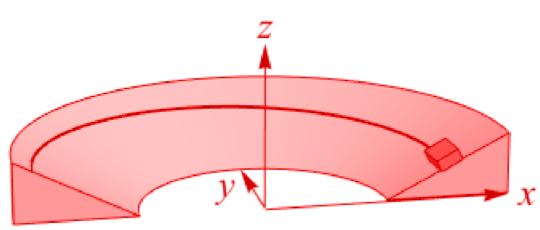
$$\left(\frac{mg}{\cos\theta - \mu_s\sin\theta}\right)\left(\sin\theta + \mu_s\cos\theta\right) = m\frac{v^2}{R} \qquad \Rightarrow \qquad v^2 = gR\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

$$v = \sqrt{gR\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)}$$

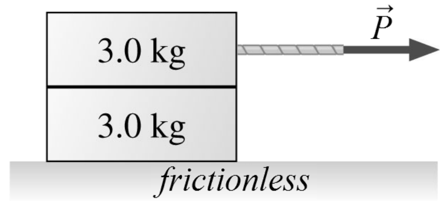
2. (6 points) Now that they've set the angle to θ, cars want to take it even faster. If there were no friction between the tires and the road, which of the following describes the path of a car going too fast?

. . . .

A car going too fast will slide to the outside of the curve.



3. (8 points) Two identical blocks are stacked, as shown. A horizontal pulling force $\vec{P}$ is applied to the top block, and the two blocks slide together across the level frictionless surface. How does the magnitude of the static friction force **from** the bottom block **on** the top block, $\vec{f}_{\mathrm{BonT}}$, compare to the magnitude of $\vec{P}$? (*On Earth.*)



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Both blocks must have the same acceleration. Considering the two blocks, together, as an "object", the pulling force $\vec{P}$ is the net external force that provides that acceleration.

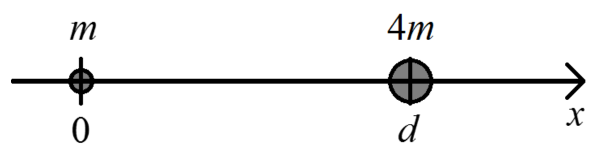
Considering the bottom block as an "object", the static friction force **from** the top block **on** the bottom block, $\vec{f}_{\mathrm{TonB}}$, is the net external force that provides that acceleration. The bottom block by itself, however, has only half the mass of the two blocks together, so the net external force required to achieve that acceleration is only half the magnitude of $\vec{P}$.

Newton's Third Law tells us that $\vec{f}_{\mathrm{BonT}}$ must have the same magnitude as $\vec{f}_{\mathrm{TonB}}$, so

$$f_{\mathrm{BonT}} = P/2$$

4. (8 points) A small sphere of mass m is placed at the origin. Another small sphere, this of mass $4m$, is placed at a position $+d$ on the x axis. At what location could a third small sphere be placed, so the net gravitational force on it was zero? *Think about this before (or even instead of) writing equations! If you do write equations, be sure to check your answer.*

. . . .

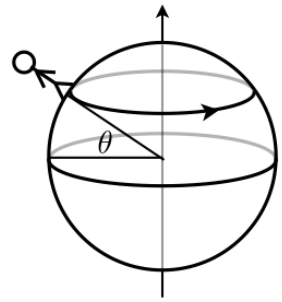


Since the gravitational force is always attractive, the new sphere must be placed between the spheres of mass m and $4m$, so it can be attracted equally to each of them.

The gravitational force follows an inverse square law, so the new sphere must be twice as far from the sphere of mass $4m$ than it is from the sphere of mass m. The two distances must add to d, so $2d/3$ and $d/3$, respectively, satisfy these condition. Place the new sphere at

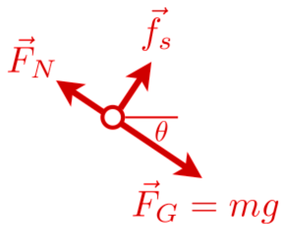
$$x = d/3$$

5. (8 points) Which of the following free body diagrams best describes a person of mass m standing on the rotating earth at θ degrees north of the equator?

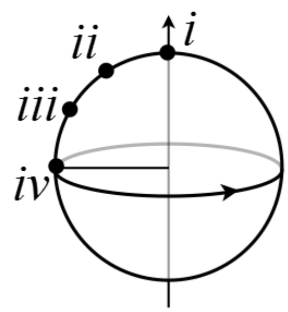


. . . .

The net force on the person must point toward the center of the path they follow. There must be a force northward, parallel to the surface (that is, a friction force), to cancel the southward component of the gravitational force.



6. (8 points) Which of the following objects of mass m have the largest apparent weight on the rotating earth?



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Consider, first, object iv on the equator. Let it's mass be m, and use Newton's Second Law. Sketch a Free Body Diagram. There will be a gravitational force mgtoward the Earth's center, and a normal force n away from the Earth's center. This normal force is the object's apparent weight. Choose an axis in the direction of the object's acceleration, toward the center of the circular path it follows as the Earth rotates. Showing signs explicitly in Newton's Second Law so symbols represent magnitudes,

$$\sum F_c = mg - n = ma_c = mr\omega^2 \qquad \Rightarrow \qquad n = mg - mr\omega^2$$

As one moves away from the equator toward the pole, the distance r from the object to the rotation axis decreases. The normal force will therefore increase. The object with greatest apparent weight will be the object at the pole ($r = 0$), which is

Object i

7. (8 points) If I weigh 883 N on earth, how much would I weigh on another planet with 6 times the mass and 2.5 times the radius?

. . . .

My weight on Earth is the Earth's gravitational force on me,

$$F_E = G\frac{M_E m}{R_E^2} = 883\,\mathrm{N}$$

where M_E is the Earth's mass, m is my mass, and R_E is the Earth's radius. On this other planet P, the gravitational force on me will be

$$F_P = G\frac{M_P m}{R_P^2} = G\frac{(6M_E)\,m}{(2.5R_E)^2} = \left(\frac{6}{2.5^2}\right)G\frac{M_E m}{R_E^2} = \left(\frac{6}{2.5^2}\right)883\,\mathrm{N} = 848\,\mathrm{N}$$