
I. (16 points) A rocket-powered car moves along a line, starting from rest and accelerating horizontally with an acceleration a that depends on time t according to

$$a(t) = \left(-12\,\mathrm{m/s^4}\right) t^2 + \left(48\,\mathrm{m/s^3}\right) t$$

until the acceleration reaches zero. After that point the car cruises at a constant speed. What is that speed?

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Let

$$A = -12\,\mathrm{m/s^4} \qquad \text{and} \qquad B = 48\,\mathrm{m/s^3}$$

The acceleration reaches zero at

$$At^2 + Bt = 0 \qquad \Rightarrow \qquad (At + B)\,t = 0$$

so

$$t = 0 \qquad \text{or} \qquad At + B = 0 \qquad \Rightarrow \qquad t = \frac{-B}{A} = \frac{-\left(48\,\mathrm{m/s^3}\right)}{-12\,\mathrm{m/s^4}} = 4.0\,\mathrm{s}$$

Then in one dimension

$$a = \frac{dv}{dt} \qquad \Rightarrow \qquad v = \int a\,dt = \int \left(At^2 + Bt\right) dt = \frac{A}{3}t^3 + \frac{B}{2}t^2 + C$$

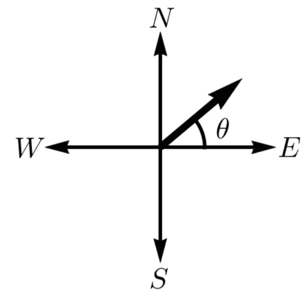
where C is an integration constant. The car starts at rest, so

$$v(0) = \frac{A}{3}0^3 + \frac{B}{2}0^2 + C = 0 \qquad \Rightarrow \qquad C = 0$$

So

$$v(4.0\,\mathrm{s}) = \frac{-12\,\mathrm{m/s^4}}{3}\left(4.0\,\mathrm{s}\right)^3 + \frac{48\,\mathrm{m/s^3}}{2}\left(4.0\,\mathrm{s}\right)^2 = -256\,\mathrm{m/s} + 384\,\mathrm{m/s} = 130\,\mathrm{m/s}$$

II. (16 points) Some scientists are exploring Mars. Their rover is at Checkpoint A, 6.5 miles east and 4.6 miles north of their settlement. From that Checkpoint, they begin heading 45° north of east to collect more samples. After they have gone 1.5 miles, they realize that they only have enough fuel remaining to travel 8.2 miles. They turn their rover around and head directly towards the settlement. How far from camp are they when they run out of fuel?



When the scientists turn around, they are

$$6.5\,\text{miles} + (1.5\,\text{miles})\cos 45^\circ = 7.56\,\text{miles} \quad \text{East}$$

and

$$4.6\,\text{miles} + (1.5\,\text{miles})\sin 45^\circ = 5.66\,\text{miles} \quad \text{North}$$

of the settlement. Using the Pythagorean theorem, they are

$$\sqrt{(7.56\,\text{miles})^2 + (5.66\,\text{miles})^2} = 9.44\,\text{miles}$$

away from the settlement. If they run out of fuel after traveling 8.2 miles, they'll need to walk

$$9.44\,\text{miles} - 8.2\,\text{miles} = 1.2\,\text{miles}$$

1. (6 points) At what angle (measured North of East) must they to drive to get back?

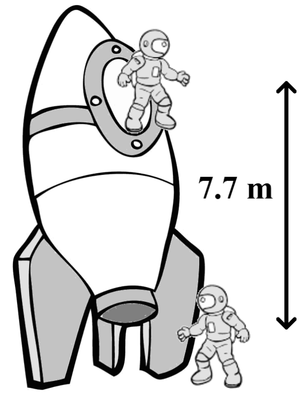
The direction (North of East) *from* the settlement *to* the point where they turned around is

$$\theta = \tan^{-1}\left(\frac{5.66\,\text{miles}}{7.56\,\text{miles}}\right) = 37^\circ$$

The scientists must travel in the opposite direction (that is, *from* the point where they turned around *to* the settlement), or at

$$37^\circ + 180^\circ = 217^\circ \approx 218^\circ$$

III. (16 points) An astronaut on Venus throws a geologic sample straight up to her companion in the ship 7.7 m above her. The companion misses the sample on its way up, but catches it on its way down. If the sample is thrown at 11.9 m/s, and is traveling at 2.3 m/s when caught, how much time elapses between the throw and the catch? (*NOT on Earth!*)



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The sample has a constant acceleration, so the problem can be solved using constant-acceleration kinematics. Choose a coordinate system. I'll choose the origin at the launch point of the sample, with positive y upward. Therefore

$$y = y_0 + v_0\,\Delta t + \tfrac{1}{2}a\,(\Delta t)^2 \qquad \text{and} \qquad v = v_0 + a\,\Delta t$$

With this choice of coordinate system,

$$y = +7.7\,\mathrm{m} \qquad y_0 = 0\,\mathrm{m} \qquad v_0 = +11.9\,\mathrm{m/s} \qquad v = -2.3\,\mathrm{m/s}$$

The acceleration is neither known nor asked, so it should be eliminated.

$$a = \frac{v - v_0}{\Delta t} \qquad \text{so} \qquad y = y_0 + v_0\,\Delta t + \tfrac{1}{2}\left(\frac{v - v_0}{\Delta t}\right)(\Delta t)^2 = y_0 + v_0\,\Delta t + \tfrac{1}{2}\,(v - v_0)\,\Delta t = y_0 + \tfrac{1}{2}\,(v + v_0)\,\Delta t$$

Solve for time.

$$\Delta t = \frac{2\,(y - y_0)}{v + v_0} = \frac{2\,(7.7\,\mathrm{m} - 0\,\mathrm{m})}{-2.3\,\mathrm{m/s} + 11.9\,\mathrm{m/s}} = 1.6\,\mathrm{s}$$

2. (6 points) If the astronaut in the spaceship now wants to throw a box of plastic bags down to the astronaut on the surface of Venus, with what initial velocity should they throw the box so that they will have a speed of 11.9 m/s when the astronaut on the ground catches them?

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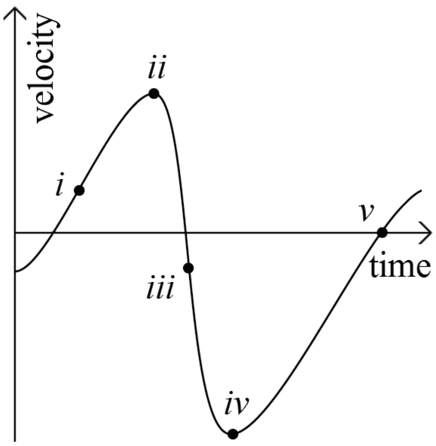
Since the astronaut on the ground threw the sample at 11.9 m/s and it was caught at 2.3 m/s, if astronaut on the ship throws the bundle of bags at 2.3 m/s, they will be caught at 11.9 m/s. If the bags are thrown upward, they'll rise, stop, and fall, passing by the upper astronaut at 2.3 m/s. So

They could throw them up or down at 2.3 m/s.

3. (8 points) An object moves on a non-linear spring such that its velocity varies with time as shown. At what point is the magnitude of acceleration the greatest?



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As $a = dv/dt$, the greatest acceleration magnitude will be when the velocity-time curve is steepest, or point

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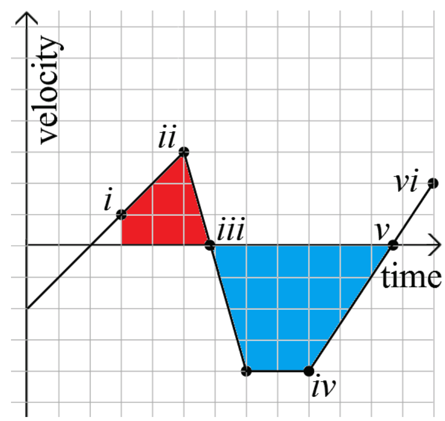
4. (8 points) An object moves with a velocity that varies with time as shown. It passes the origin at the point i. At what point is it farthest from the origin?



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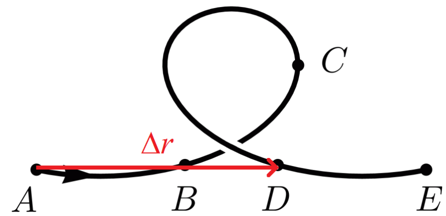
As

$$\Delta x = \int v \, dt$$

the area under the velocity-time curve represents displacement. From i to iii the object moves a distance represented by a bit more than 5 "squares" from the origin in the positive direction. From iii to v it moves a distance represented by about $15\frac{1}{2}$ squares in the negative direction. The first 5+ square of this just brings the object back to the origin. But as it continues in the negative direction for another 10+ squares, it is farther from the origin at v than it was at iii. From v to vi, the object moves a square and a half closer to the origin again, so its greatest distance from the origin occurred

At v.

5. (8 points) A bee flies at constant speed from point A to point E along the path shown at the right. Which of the vectors below best corresponds to the direction of the average *velocity* between the points A and D?



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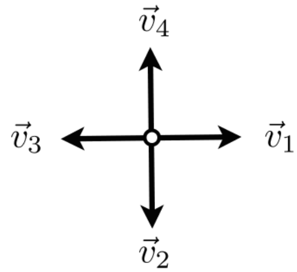
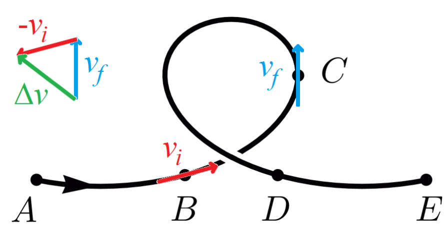
The displacement $\Delta\vec{r}$ of the bee from A to D is to the right, as shown. As $\vec{v}_{\text{avg}} = \Delta\vec{r}/\Delta t$, the average velocity is in the same direction as the displacement, which is

$\vec{v}_1$



6. (8 points) A bee flies at constant speed from point A to point E along the path shown at the right. Which of the vectors below best corresponds to the direction of the average *acceleration* between the points B and C?



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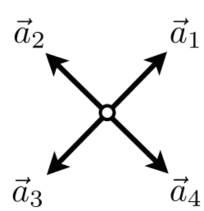
Vectors representing the initial velocity at A and the final velocity at C are shown on the bee's path. A triangle representing $\Delta\vec{v} = \vec{v}_f + (-\vec{v}_i)$ is also shown. As $\vec{a}_{\text{avg}} = \Delta\vec{v}/\Delta t$, the average acceleration is in the same direction as the velocity change, which is

$\vec{a}_2$



7. (8 points) A rubber ball is dropped and bounces from the floor. A motion diagram shows the ball released from rest at time 1, and in contact with the floor at time 4. Note that the diagram for upward motion has been offset for clarity. At which time(s), if any, does the average acceleration have the greatest magnitude?

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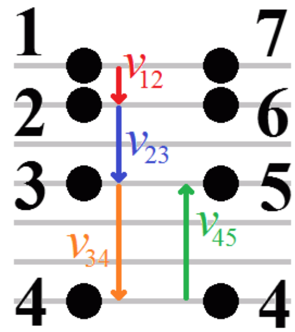

Consider the average acceleration at time 2. The average velocity from 1 to 2 is shown, proportional to the displacement from 1 to 2, as $\vec{v}_{\text{avg}} = \Delta\vec{r}/\Delta t$. The average velocity from 2 to 3 is also shown, proportional to the displacement from 2 to 3. As $\vec{a}_{\text{avg}} = \Delta\vec{v}/\Delta t$, the average acceleration is proportional to the velocity change $\vec{v}_{23} + (-\vec{v}_{12})$. In terms of the horizontal bar spacing, $\vec{v}_{23}$ is two spaces down, and $\vec{v}_{12}$ is one space down. The difference, one space down, is proportional to the average acceleration.

While the ball is in free fall (moving under the influence of only gravity), the acceleration is constant, so "one space down" represents the acceleration at times 1, 2, 3, 5, 6, and 7. However, the ball is in contact with the floor at time 4, and so is **not** in free fall. The average acceleration at that time must be determined, to discover whether it is greater or less than "one space down".

The average acceleration at time 4 is proportional to the velocity change $\vec{v}_{45} + (-\vec{v}_{34})$. In terms of the horizontal bar spacing, $\vec{v}_{45}$ is three spaces up, and $\vec{v}_{34}$ is three spaces down. The difference, six space up, is proportional to the average acceleration at time 4, and is of much greater magnitude than the acceleration while the ball is in free fall. The greatest acceleration magnitude occurs

At time 4.