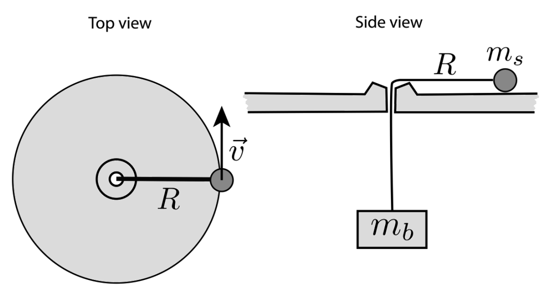
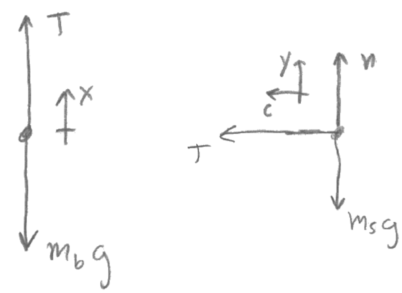
I. (16 points) A block of mass m_b is suspended vertically on a ideal cord that then passes through a frictionless hole and is attached to a sphere of mass m_s, which is rotating on a frictionless flat surface. If the sphere moves in a circle of radius R, the block will eventually come to rest while hanging from the cord. At this equilibrium point, what is the speed, v, of the sphere in terms of the masses of the two objects, the radius R, and g? (*On Earth.*)



. . . .

Use Newton's' Second Law. Sketch a Free Body Diagram of the block. It will have a tension T upward, and a gravitational force $m_b g$ downward. Choose a coordinate system (I'll choose positive x upward). Write Newton's' Second Law for that axis. I'll show signs explicitly, so symbols represent magnitudes. Note that the block is not accelerating.

$$\sum F_x = T - m_b g = m_b a_x = 0 \qquad \Rightarrow \qquad T = m_b g$$



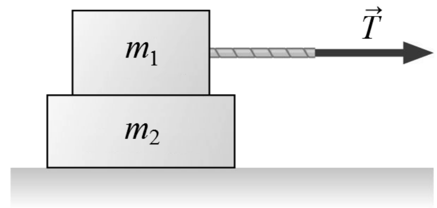
Now sketch a Free Body Diagram of the sphere. It will have a tension T toward the center of its circular path, a gravitational force $m_s g$ downward, and a normal force n upward. Choose a coordinate system (I'll choose positive c toward the center). Write Newton's' Second Law for that axis. Again, I'll show signs explicitly, so symbols represent magnitudes. Note that the sphere is accelerating toward the center of its circular path.

$$\sum F_x = T = m_s a_c = m_s \frac{v^2}{R}$$

Substitute the expression previously found for T, and solve for v.

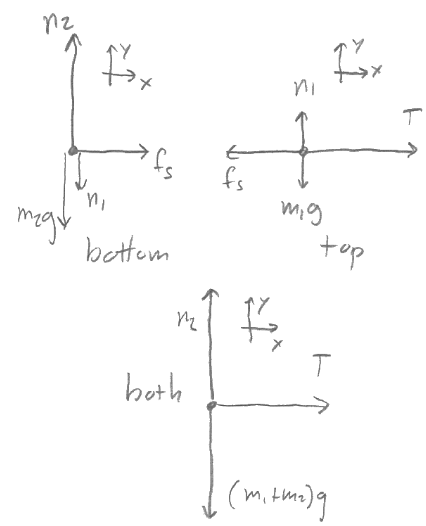
$$m_b g = m_s \frac{v^2}{R} \qquad \Rightarrow \qquad v = \sqrt{gRm_b/m_s}$$

II. (16 points) Two blocks, with masses m_1 and m_2, are stacked on a frictionless surface with m_1 above m_2, as shown. The coefficient of static friction between the blocks is μ_s and the coefficient of kinetic friction is μ_k. With what maximum tension T_top may the rope attached to the top block be pulled, if the top block is not to slide on the bottom block? Express your answer in terms of parameters defined in the problem, and physical or mathematical constants. (*On Earth.*)



. . . .

Since the only horizontal force on the bottom block is the force of static friction, let us first find the maximum acceleration of the bottom block.



Use Newton's' Second Law. Sketch a Free Body Diagram of the bottom block. It will have a normal force n_2 upward from the surface, a gravitational force $m_2 g$ downward, a normal force n_1 downward from the top block, and a friction force f_s to the right. Choose a coordinate system (I'll choose positive x to the right and positive y upward). Write Newton's' Second Law for the x axis. I'll show signs explicitly, so symbols represent magnitudes. Note that the block is not accelerating.

$$\sum F_x = f_\mathrm{s} = m_2 a_x \qquad \Rightarrow \qquad a_x = f_\mathrm{s}/m_2 = \mu_\mathrm{s} n_1/m_2$$

where the static friction force has its maximum value, as we're looking for the maximum acceleration.

Now apply Newton's Second Law to the top block, to find an expression for n_1. Sketch a Free Body Diagram. There will be a normal force n_1 upward, a gravitational force $m_1 g$ downward, a tension force T to the right, and a static friction force f_s to the left. With the same x and y axes as chosen for the bottom block, write Newton's Second Law for the y axis.

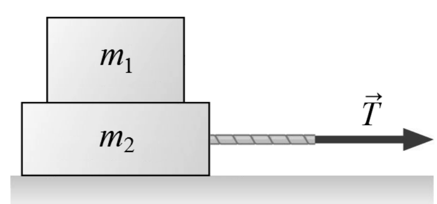
$$\sum F_y = n_1 - m_1 g = m_1 a_y = 0 \qquad \Rightarrow \qquad n_1 = m_1 g$$

where the top block isn't accelerating vertically.

Now consider the two blocks together as one object. Sketch a Free Body Diagram. There will be a normal force n_2 upward, a gravitational force $m_1 + m_2$ downward, and a tension force T to the right. Note that the acceleration of this two-block object must be the same as the acceleration of each individual block. Substitute the expression found for n_1.

$$\sum F_x = T = (m_1 + m_2)\, a_x = (m_1 + m_2)\, \mu_\mathrm{s} n_1/m_2 = (m_1 + m_2)\, \frac{\mu_\mathrm{s} m_1}{m_2} g$$

1. (6 points) If the rope is, instead, attached to the bottom block, how does the new maximum force T_bottom with which it can be pulled, if the top block is not to slide on the bottom block, compare to the value for T_top found above? (Hint: consider the friction force between the blocks in the two situations.)

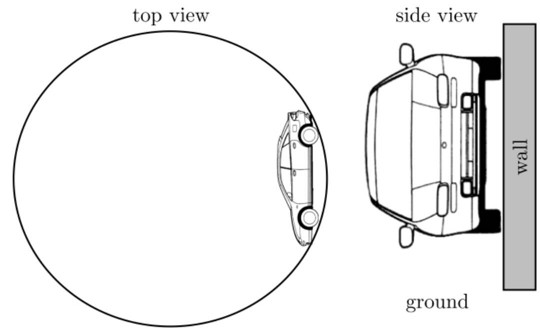


. . . .

The maximum force of static friction is the same, regardless of which block the rope is attached to. That is the only force accelerating whichever block the rope is not attached to. The more mass that block has, the lower its maximum acceleration can be. Since the total mass of the two blocks is constant, if the maximum acceleration is lower, the maximum tension must be lower as well. So the maximum tension must be lower if it is attached to the block with less mass.

$T_\mathrm{bottom} > T_\mathrm{top}$ if $m_2 > m_1$, but $T_\mathrm{bottom} < T_\mathrm{top}$ if $m_2 < m_1$.

III. (16 points) A 525 kg car is about to attempt the stunt the Wall of Death, where it drives around a vertical wall in a circular track of radius 21 m. The coefficients of static and kinetic friction between the tires and the wall are $\mu_s = 0.75$ and $\mu_k = 0.60$, respectively. What is the minimum speed the car must maintain to stay on the wall? (*On Earth.*)



. . . .

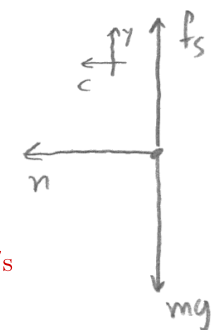
Use Newton's' Second Law. Sketch a Free Body Diagram of the car, from the side view. It will have a normal force n to the left, a gravitational force mg downward, and a friction force f_s upward. Choose a coordinate system (I'll choose positive c to the left and positive y upward). Write Newton's' Second Law for the y axis. I'll show signs explicitly, so symbols represent magnitudes. Note that the car is not accelerating vertically.

$$\sum F_y = f_s - mg = ma_y = 0 \qquad \Rightarrow \qquad \mu_s n = mg$$

where the static friction force has its maximum value, as we're looking for the minimum speed.

Now write Newton's Second Law for the c axis.

$$\sum F_c = n = ma_c = m\frac{v^2}{R}$$



Substitute this expression for n into the result from the y axis, and solve for v.

$$\mu_s\left(m\frac{v^2}{R}\right) = mg \qquad \Rightarrow \qquad v = \sqrt{\frac{gR}{\mu_s}} = \sqrt{\frac{(9.81\,\text{m/s}^2)\,(21\,\text{m})}{0.75}} = 17\,\text{m/s}$$

2. (6 points) If the car is traveling around the track at 28 m/s, what is the magnitude of the apparent weight felt by the 75 kg driver?

. . . .

The apparent weight is the magnitude of the force supporting the driver. That force comes from the driver's seat, and has components both perpendicular (normal) and parallel (friction) to the surface. Using the same axes chosen above,

$$\sum F_c = n = ma_c = m\frac{v^2}{R} = (75\,\text{kg})\,\frac{(28\,\text{m/s})^2}{21\,\text{m}} = 2800\,\text{N}$$

and

$$\sum F_y = f_s - mg = ma_y = 0 \qquad \Rightarrow \qquad f_s = mg = (75\,\text{kg})\,\left(9.81\,\text{m/s}^2\right) = 736\,\text{N}$$

so

$$F_{\text{seat}} = \sqrt{n^2 + f_s^2} = \sqrt{(2800\,\text{N})^2 + (736\,\text{N})^2} = 2895\,\text{N}$$

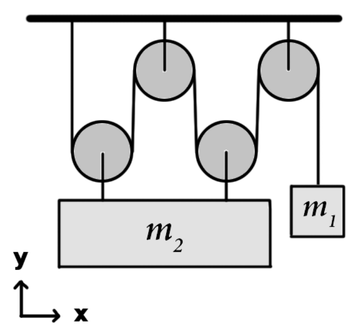
3. (8 points) In a safely-operating Ferris wheel, passengers never leave their seats. Consider passengers in the car at the very top of the rotating wheel. A normal force from the seat supports them. What force or forces, if any, make a Newton's Third Law force pair with that normal force?

. . . .

Both forces in a force pair must be the same kind of force, and must be between the same two objects. Therefore, the force paired with that of the seat on the passengers must be . . .

The normal force from the passengers on the seat.

4. (8 points) A pulley system with two objects m_1 and $m_2 = 3m_1$ is shown. m_1 is released from rest. After releasing, the y components of the acceleration for m_1 and m_2 are a_1 and a_2. What is the sign of a_2 in the given coordinate system, and how does its magnitude compare to a_1?



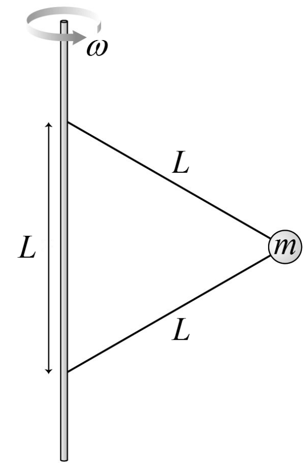
. . . .

If object m_1 moves a distance d in a particular time, the object m_2 can only move one-fourth as far. Each of the four segments of rope supporting m_2 will change their length by $d/4$. That is sufficient to choose

$$a_2 > 0 \quad \text{and} \quad |a_2| = |a_1|/4$$

from among the choices offered. However, you will also note that if the tension in the rope is T, just T supports the object m_1, but $4T$ supports the object $m_2 = 3m_1$. As $4T/3m_1 > T/m_1$, the object m_1 falls and the object m_2 rises, so $a_2 > 0$.

5. (8 points) Two strings of length L are tied to the sphere of mass m, as shown. The whole apparatus spins with constant angular speed ω. Is it possible for both strings to exert a tension force? If so, how do the tensions in the upper and lower strings compare? (*On Earth.*)



. . . .

As the sphere moves in a circle, the horizontal component of the tensions provides the force accelerating it toward the center. At slow speeds, less force is required, so the lower string may go slack. But it is possible to put both strings into tension by spinning the apparatus fast enough.

The tension of the upper string must have sufficient upward component to balance both the downward component of the lower string's tension, *and* the downward gravitational force on the sphere.

It **is** possible for both strings to exert a tension force.
The upper string must have **greater** tension than the lower string.

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6. (8 points) A satellite is orbiting a planet of radius R and mass $m_1 = M$ at an altitude of h meters and at speed v_1. An identical satellite is orbiting a different planet, also of radius R, but with mass $m_2 = 2M$. If this second satellite is also at an altitude of h, how does the orbital speed of the second satellite v_2 compare to the orbital speed of the first satellite v_1?

. . . .

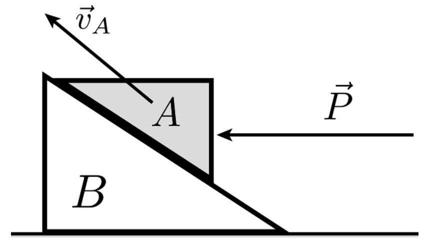
Use Newton's Second Law. Choose an axis that points toward the center of the satellite's circular path. The only force on the satellite is that of gravity.

$$\sum F_c = F_G = ma_c = m\frac{v^2}{r} \qquad \Rightarrow \qquad G\frac{m_{\text{p}}m}{r^2} = m\frac{v^2}{r}$$

where m_{p} is the mass of a planet. Remembering that the radii of the two orbits are the same ($r = R + h$),

$$v^2 = Gm_{\text{p}}/r \qquad \Rightarrow \qquad \left(\frac{v_2}{v_2}\right)^2 = \frac{Gm_2/r}{Gm_1/r} = \frac{m_2}{m_1} = \frac{2M}{M} = 2 \qquad \Rightarrow \qquad v_2 = v_1\sqrt{2}$$

7. (8 points) Two slanted blocks are placed on top of each other and the top block (A) is given a push to the left. As a result, block A slides up the lower block (B). However, block B is resting on the Earth and does not move. Which of the free body diagrams below is the best Free Body Diagram for **block B**? (Hint. Draw a FBD for block A and use interaction pairs).



. . . .

The normal force of A on B must be perpendicular to the surface between them. The friction force of A on B must be parallel to the surface between them.